Exercise 1.2

Unit 1 · Real Numbers

1 **By using the product and quotient rules for radicals, write each expression as a single radical and simplify.**

(i) $\sqrt[3]{6} \cdot \sqrt[3]{6}$

$= \sqrt[3]{6 \times 6}$

$= \sqrt[3]{36}$

$\therefore = \sqrt[3]{36}$

(ii) $\sqrt[5]{4} \cdot \sqrt[5]{8}$

$= \sqrt[5]{4 \times 8} = \sqrt[5]{32}$

$= \sqrt[5]{2^5} = 2$

$\therefore = 2$

(iii) $\sqrt[4]{x} \cdot \sqrt[4]{x^3}$

$= \sqrt[4]{x \cdot x^3}$

$= \sqrt[4]{x^4} = x$

$\therefore = x$

(iv) $\sqrt{10} \cdot \sqrt[3]{11}$

$\therefore$ Product rule does not apply

The product rule for radicals does not apply, because the indices (2 and 3) are not the same.

(v) $\dfrac{\sqrt[4]{x^7}}{\sqrt[4]{x^5}}$

$= \sqrt[4]{\dfrac{x^7}{x^5}} = \sqrt[4]{x^{7-5}}$

$= \sqrt[4]{x^2} = (x^2)^{\frac{1}{4}} = x^{\frac{1}{2}} = \sqrt{x}$

$\therefore = \sqrt{x}$

(vi) $\dfrac{\sqrt[3]{5000}}{\sqrt[3]{5}}$

$= \sqrt[3]{\dfrac{5 \times 1000}{5}}$

$= \sqrt[3]{1000} = \sqrt[3]{10^3} = 10$

$\therefore = 10$

(vii) $\dfrac{\sqrt{500}}{\sqrt{5}}$

$= \sqrt{\dfrac{5 \times 100}{5}}$

$= \sqrt{100} = 10$

$\therefore = 10$

(viii) $\sqrt{10} \cdot \sqrt[3]{7}$

$\therefore$ Product rule does not apply

The product rule for radicals does not apply, because the indices (2 and 3) are not the same.

2 Write each exponential expression as an equivalent radical expression and simplify if possible.

(i) $(216)^{\frac{2}{3}}$

$= \left(216^2\right)^{\frac{1}{3}} = \sqrt[3]{216^2}$

$= \sqrt[3]{(2^3 \cdot 3^3)^2}$

$= \sqrt[3]{(2^2 \cdot 3^2)^3}$

$= 2^2 \cdot 3^2 = 4 \times 9 = 36$

$\therefore = 36$

(ii) $(29)^{\frac{1}{2}}$

$= \sqrt{29}$

$\therefore = \sqrt{29}$

(iii) $\left(\dfrac{1}{32}\right)^{\frac{1}{5}}$

$= \sqrt[5]{\dfrac{1}{32}} = \sqrt[5]{\dfrac{1}{2^5}}$

$= \dfrac{\sqrt[5]{1}}{\sqrt[5]{2^5}} = \dfrac{1}{2}$

$\therefore = \dfrac{1}{2}$

(iv) $(216)^{\frac{-2}{3}}$

$= \frac{1}{(216)^{\frac{2}{3}}} = \frac{1}{(216^2)^{\frac{1}{3}}}$

$= \frac{1}{\sqrt[3]{216^2}} = \frac{1}{\sqrt[3]{(2^3 \cdot 3^3)^2}}$

$= \frac{1}{\sqrt[3]{((2 \cdot 3)^2)^3}} = \frac{1}{6^2} = \frac{1}{36}$

$\therefore = \frac{1}{36}$

(v) $(1000)^{\frac{1}{3}}$

$= \sqrt[3]{1000}$

$= \sqrt[3]{10^3} = 10$

$\therefore = 10$

(vi) $\left(\frac{1}{39}\right)^{\frac{1}{2}}$

$= \sqrt{\frac{1}{39}} = \frac{\sqrt{1}}{\sqrt{39}}$

$= \frac{1}{\sqrt{39}}$

$\therefore = \frac{1}{\sqrt{39}}$

3 **Write each radical expression as an equivalent exponential expression and simplify if possible.**

(i) $\left(\sqrt[3]{5}\right)^2$

$= \left(5^{\frac{1}{3}}\right)^2$

$= 5^{\frac{2}{3}}$ or $(25)^{\frac{1}{3}}$

$\therefore = 5^{\frac{2}{3}}$

(ii) $\left(\sqrt[4]{10}\right)^8$

$= \left((10)^{\frac{1}{4}}\right)^8$

$= 10^2 = 100$

$\therefore = 100$

(iii) $-\left(\sqrt[3]{6}\right)^6$

$= -\left((6)^{\frac{1}{3}}\right)^6$

$= -6^2 = -36$

$\therefore = -36$

(iv) $\left(\sqrt[3]{6}\right)^6$

$= \left((6)^{\frac{1}{3}}\right)^6$

$= 6^2 = 36$

$\therefore = 36$

(v) $-\left(\sqrt[3]{5}\right)^2$

$= -\left((5)^{\frac{1}{3}}\right)^2$

$= -5^{\frac{2}{3}}$ or $-(25)^{\frac{1}{3}}$

$\therefore = -5^{\frac{2}{3}}$

(vi) $-\left(\sqrt[4]{10}\right)^8$

$= -\left((10)^{\frac{1}{4}}\right)^8$

$= -10^2 = -100$

$\therefore = -100$

4 **Use the properties of exponents to simplify each of the following. Assume that all variables represent positive numbers. Write all results with positive exponents.**

(i) $\dfrac{16^{\frac{1}{5}} \cdot 16^{\frac{1}{4}}}{16^{\frac{-3}{10}}}$

$= \dfrac{16^{\frac{1}{5}+\frac{1}{4}}}{16^{\frac{-3}{10}}} = 16^{\frac{1}{5}+\frac{1}{4}+\frac{3}{10}}$

$= 16^{\frac{4+5+6}{20}} = 16^{\frac{15}{20}} = 16^{\frac{3}{4}}$

$= \left(2^4\right)^{\frac{3}{4}} = 2^3 = 8$

$\therefore = 8$

(ii) $7^{\frac{-1}{3}}\left(7^{\frac{5}{3}} - 7^{\frac{4}{3}}\right)$

$= 7^{\frac{-1}{3}} \cdot 7^{\frac{5}{3}} - 7^{\frac{-1}{3}} \cdot 7^{\frac{4}{3}}$

$= 7^{\frac{-1}{3}+\frac{5}{3}} - 7^{\frac{-1}{3}+\frac{4}{3}}$

$= 7^{\frac{4}{3}} - 7^{\frac{3}{3}} = 7^{\frac{4}{3}} - 7$

$= 7 \cdot 7^{\frac{1}{3}} - 7 = 7\left(7^{\frac{1}{3}} - 1\right)$

$\therefore = 7\left(7^{\frac{1}{3}} - 1\right)$

(iii) $\dfrac{2^{\frac{2}{3}} \cdot 2^{\frac{1}{7}}}{2^{\frac{1}{2}}}$

$= \dfrac{2^{\frac{2}{3}+\frac{1}{7}}}{2^{\frac{1}{2}}} = 2^{\frac{2}{3}+\frac{1}{7}-\frac{1}{2}}$

$= 2^{\frac{28+6-21}{42}} = 2^{\frac{13}{42}}$

$\therefore = 2^{\frac{13}{42}}$

(iv) $\dfrac{3^{\frac{-1}{2}} \cdot 3^{\frac{1}{2}}}{3^{\frac{1}{2}}}$

$= \dfrac{3^{\frac{-1}{2}+\frac{1}{2}}}{3^{\frac{1}{2}}} = \dfrac{3^0}{3^{\frac{1}{2}}} = \dfrac{1}{3^{\frac{1}{2}}}$

$\therefore = \dfrac{1}{3^{\frac{1}{2}}}$

(v) $\left(\dfrac{36^{\frac{1}{2}} \cdot 6^{\frac{1}{2}}}{8^{\frac{1}{2}} \cdot 27^{\frac{1}{2}}}\right)^3$

$= \left(\dfrac{(6^2)^{\frac{1}{2}} \cdot 6^{\frac{1}{2}}}{(2^3)^{\frac{1}{2}} \cdot (3^3)^{\frac{1}{2}}}\right)^3$

$= \left(\dfrac{6 \cdot 6^{\frac{1}{2}}}{2^{\frac{3}{2}} \cdot 3^{\frac{3}{2}}}\right)^3 = \left(\dfrac{6^{\frac{3}{2}}}{2^{\frac{3}{2}} \cdot 3^{\frac{3}{2}}}\right)^3$

$= \left(\dfrac{2^{\frac{3}{2}} \cdot 3^{\frac{3}{2}}}{2^{\frac{3}{2}} \cdot 3^{\frac{3}{2}}}\right)^3$

$= (2^0 \cdot 3^0)^3 = (1 \cdot 1)^3 = 1$

$\therefore = 1$

(vi) $\left(\frac{2187a^5b^{17}}{a^{12}b^3}\right)^{\frac{1}{7}}$

$= \left(\frac{3^7a^5b^{17}}{a^{12}b^3}\right)^{\frac{1}{7}}$ $(2187 = 3^7)$

$= \left(3^7\, a^{5-12}\, b^{17-3}\right)^{\frac{1}{7}} = \left(3^7\, a^{-7}\, b^{14}\right)^{\frac{1}{7}}$

$= \frac{(3^7)^{\frac{1}{7}}\,(b^{14})^{\frac{1}{7}}}{(a^7)^{\frac{1}{7}}} = \frac{3b^2}{a}$

$\therefore = \frac{3b^2}{a}$

(vii) $\sqrt[4]{\frac{a^3}{b^3}} \times \sqrt[4]{\frac{b^3}{c^3}} \times \sqrt[4]{\frac{c^3}{a^3}}$

$= \sqrt[4]{\frac{a^3}{b^3} \times \frac{b^3}{c^3} \times \frac{c^3}{a^3}}$

$= \sqrt[4]{a^{3-3}\, b^{3-3}\, c^{3-3}} = \sqrt[4]{a^0b^0c^0}$

$= \sqrt[4]{1} = 1$

$\therefore = 1$

(5) Use suitable laws of exponents to show that

$$\left(\frac{x^p}{x^q}\right)^{p+q} \times \left(\frac{y^q}{y^r}\right)^{q+r} \times \left(\frac{z^r}{z^p}\right)^{r+p} \times x^{q^2} \times y^{r^2} \times z^{p^2}$$
$$= x^{p^2} \times y^{q^2} \times z^{r^2}$$

Proof: Take the L.H.S.

$L.H.S. = \left(\frac{x^p}{x^q}\right)^{p+q} \times \left(\frac{y^q}{y^r}\right)^{q+r} \times \left(\frac{z^r}{z^p}\right)^{r+p} \times x^{q^2}y^{r^2}z^{p^2}$

$= \left(x^{p-q}\right)^{p+q} \times \left(y^{q-r}\right)^{q+r} \times \left(z^{r-p}\right)^{r+p} \times x^{q^2}y^{r^2}z^{p^2}$

$= x^{(p-q)(p+q)} \times y^{(q-r)(q+r)} \times z^{(r-p)(r+p)} \times x^{q^2}y^{r^2}z^{p^2}$

$= x^{p^2-q^2} \times y^{q^2-r^2} \times z^{r^2-p^2} \times x^{q^2} \times y^{r^2} \times z^{p^2}$

$= x^{p^2-q^2+q^2} \times y^{q^2-r^2+r^2} \times z^{r^2-p^2+p^2}$

$= x^{p^2} \times y^{q^2} \times z^{r^2} = R.H.S.$

$\therefore$ Hence proved