Exercise 11.2

Unit 11 · Basic Statistics

Mean: $\bar{x} = \frac{\Sigma x}{n}$ Grouped: $\bar{x} = \frac{\Sigma fx}{\Sigma f}$ Weighted: $\bar{x}_w = \frac{\Sigma wx}{\Sigma w}$

Median: n odd $\rightarrow \left(\frac{n+1}{2}\right)^{th}$ value

n even $\rightarrow \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left(\frac{n+2}{2}\right)^{th} \text{ value}\right]$

Grouped median: $\tilde{x} = l + \frac{h}{f}\left(\frac{n}{2} - c\right)$

Grouped mode $= l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$

1 **Find the arithmetic mean of each set of data.**

(i) $\bar{x} = \frac{\Sigma x}{n} = \frac{2 + 4 + 6 + 8 + 10 + 12}{6}$

$= \frac{42}{6}$

$\therefore \bar{x} = 7$

(ii) $\bar{y} = \frac{\Sigma y}{n} = \frac{3 + 4 - 1 + 7 - 8 - 5 + 0}{7}$

$= \frac{0}{7}$

$\therefore \bar{y} = 0$

(iii) $\bar{z} = \frac{\Sigma z}{n} = \frac{0 + 4 + 8 + 12 + 16 + 20 + 24 + 28}{8}$

$= \frac{112}{8}$

$\therefore \bar{z} = 14$

(iv) $\bar{u} = \frac{\Sigma u}{n} = \frac{3.1 + 4.2 + 5.3 + 6.4 + 7.5 + 8.6 + 9.7 + 10.8}{8}$

$= \frac{55.6}{8}$

$\therefore \bar{u} = 6.95$

(v) $\bar{v} = \frac{\Sigma v}{n} = \frac{5 + 5 + 5 + 5 + 5 + 5 + 5 + 5}{8}$

$= \frac{40}{8}$

$\therefore \bar{v} = 5$

2 **Which of two players, A and B, is the better run-getter over 10 innings?**

$\bar{x}_A = \frac{12 + 15 + 6 + 73 + 7 + 19 + 199 + 36 + 84 + 29}{10}$

$= \frac{480}{10} = 48$

$\bar{x}_B = \frac{47 + 12 + 76 + 48 + 4 + 51 + 37 + 48 + 13 + 0}{10}$

$= \frac{336}{10} = 33.6$

$48 > 33.6$

$\therefore$ Player A is the better run-getter (higher average).

3 **Find the weighted mean of the marks of each student (weights 4, 3, 2, 1).**

Hanzala: $\bar{x}_w = \frac{70 \times 4 + 95 \times 3 + 80 \times 2 + 55 \times 1}{4 + 3 + 2 + 1} = \frac{780}{10}$

$\therefore 78$

Musa: $\bar{x}_w = \frac{80 \times 4 + 60 \times 3 + 65 \times 2 + 45 \times 1}{4 + 3 + 2 + 1} = \frac{675}{10}$

$\therefore 67.5$

Zaki: $\bar{x}_w = \frac{90 \times 4 + 55 \times 3 + 50 \times 2 + 65 \times 1}{4 + 3 + 2 + 1} = \frac{690}{10}$

$\therefore$ 69

Saleh: $\bar{x}_w = \frac{95 \times 4 + 70 \times 3 + 60 \times 2 + 60 \times 1}{4 + 3 + 2 + 1} = \frac{770}{10}$

$\therefore$ 77

Hanzala has the highest weighted mean (78).

4 **Find the mean of the grouped data.**

Class interval	f	Class mark (x)	fx
0–5	5	2.5	12.5
5–10	7	7.5	52.5
10–15	10	12.5	125
15–20	6	17.5	105
20–25	2	22.5	45
	$\Sigma f = 30$		**$\Sigma fx = 340$**

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{340}{30} = 11.33$$

$\therefore \bar{x} = 11.33$

5 **Find the mean mass of the packets.**

Classes (kg)	No. of packets (f)	Class mark (x)	fx
1–9	10	5	50
10–18	15	14	210
19–27	18	23	414
28–36	15	32	480
37–45	12	41	492
	$\Sigma f = 70$		**$\Sigma fx = 1646$**

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1646}{70} = 23.51$$

$\therefore \bar{x} = 23.51$ kg

6 **Find the median.**

(i) 1, 4, 2, 5, 3, 7, 6

Ascending order: 1, 2, 3, 4, 5, 6, 7, $n = 7$ (odd)

$$\text{Median} = \left(\frac{7+1}{2}\right)^{th} = 4^{th} \text{ value}$$

$\therefore$ Median = 4

(ii) $\pm 2, \pm 4, \pm 6, \pm 8$

Ascending order: $-8, -6, -4, -2, 2, 4, 6, 8$, $n = 8$ (even)

$$\text{Median} = \frac{1}{2}(4^{th} + 5^{th} \text{ value}) = \frac{1}{2}(-2 + 2)$$

$\therefore$ Median = 0

(iii) 0, 1, -2, -3, 4, 5, 6, 3

Ascending order: $-3, -2, 0, 1, 3, 4, 5, 6$, $n = 8$ (even)

$$\text{Median} = \frac{1}{2}(4^{th} + 5^{th} \text{ value}) = \frac{1}{2}(1 + 3)$$

$\therefore$ Median = 2

(iv) 4, 3, 1, -3, 2, -3, 3, 4, 1

Ascending order: $-3, -3, 1, 1, 2, 3, 3, 4, 4$, $n = 9$ (odd)

$$\text{Median} = \left(\frac{9+1}{2}\right)^{th} = 5^{th} \text{ value}$$

$\therefore$ Median = 2

(v) 4, 4, 4, 4, 4, 4

Ascending order: 4, 4, 4, 4, 4, 4, $n = 6$ (even)

$$\text{Median} = \frac{1}{2}(3^{th} + 4^{th} \text{ value}) = \frac{1}{2}(4 + 4)$$

$\therefore$ Median = 4

7 **Find the median radius of the circles.**

Radius (cm)	No. of circles (f)	Cumulative frequency
1.8	6	6
1.6	9	15
1.4	15	30
1.2	24	54
1.0	33	**87** ◀ median
0.8	20	107
0.6	13	120
0.4	7	127
0.2	5	132
	$\Sigma f = 132$	

$n = 132$ (even)

$\text{Median} = \frac{1}{2}(66^{th} + 67^{th} \text{ value})$

The 55th to 87th values are all 1.0.

$\text{Median} = \frac{1}{2}(1.0 + 1.0)$

$\therefore$ Median radius = 1.0 cm

8 **Find the median of the grouped data.**

Class interval	f	Cumulative frequency
40–50	3	3
50–60	8	11
60–70	11	**22** ◀ median group
70–80	5	27
80–90	4	31
	$\Sigma f = 31$	

$n = 31, \quad \frac{n}{2} = 15.5$

The 16th value lies in 60–70, the median group.

$l = 60, \quad h = 10, \quad f = 11, \quad c = 11$

c = cumulative frequency of the class before the median group

$$\tilde{x} = l + \frac{h}{f}\left(\frac{n}{2} - c\right) = 60 + \frac{10}{11}(15.5 - 11) = 60 + 4.09$$

$\therefore$ Median $\tilde{x} = 64.09$

9 **Find the mode.**

(i) 1, 2, 3, 4, 5, 6

Every value occurs once.

$\therefore$ No mode

(ii) 2, 4, 2, 3, 2, 5, 3, 2, 5, 4, 2

2 occurs 5 times (most often).

$\therefore$ Mode = 2

(iii) 120, 130, 140, 225, 125, 225, 120

120 and 225 each occur twice.

$\therefore$ Mode = 120 and 225 (bimodal)

(iv) 2, 4, 3, 5, 5, 3, 4, 2

Every value occurs the same number of times (twice).

$\therefore$ No mode

10 **Find the mean, median and mode of the data.**

x	f	fx	c.f.
0	3	0	3
1	5	5	8
2	9	18	17
3	4	12	21
4	2	8	23
5	1	5	24
	$\Sigma f = 24$	$\Sigma fx = 48$	

Mean: $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{48}{24} = 2$

$\therefore$ Mean = 2

Median: $n = 24$, Median $= \frac{1}{2}(12^{th} + 13^{th}$ value$)$

The 9th to 17th values are 2, so Median $= \frac{1}{2}(2 + 2) = 2$

$\therefore$ Median = 2

Mode: 2 has the highest frequency (9).

$\therefore$ Mode = 2

11 Find the mode of the grouped data.

(a)

Classes	f	Class boundaries
1–4	9	0.5–4.5
5–8	8	4.5–8.5
9–12	**10** ◀ modal	8.5–12.5
13–16	6	12.5–16.5
17–20	7	16.5–20.5
	$\Sigma f = 40$	

Largest frequency $f_m = 10$, so the modal group is 8.5–12.5.

$l = 8.5$, $h = 4$, $f_m = 10$, $f_1 = 8$, $f_2 = 6$

$$\text{Mode} = 8.5 + \frac{10 - 8}{2(10) - 8 - 6} \times 4 = 8.5 + \frac{2}{6} \times 4 = 8.5 + 1.33$$

$\therefore$ Mode = 9.83

(b)

Classes	f	
10–15	7	◀ modal
15–20	6	
20–25	3	
25–30	2	
30–35	1	

Largest frequency $f_m = 7$, so the modal group is 10–15.

It is the first class, so $f_1 = 0$.

$l = 10, \quad h = 5, \quad f_m = 7, \quad f_1 = 0, \quad f_2 = 6$

$$\text{Mode} = 10 + \frac{7 - 0}{2(7) - 0 - 6} \times 5 = 10 + \frac{7}{8} \times 5 = 10 + 4.375$$

$\therefore$ Mode $= 14.375 \approx 14.38$