ExamPath

CLASS 9 MATHEMATICS · FBISE · UNIT 2 · SECTIONS B AND C PRACTICE

Unit 2: Logarithms: short and long questions

Write your working in the boxes. Model answers are on the last pages. These are practice questions written by ExamPath, not official FBISE questions or marking schemes.

Section B style: short questions (3 marks each)

B1 · 3 marks · Scientific notation

Write in scientific notation: (a) 0.000 703 (b) 38 400 000. (c) Write 2.5×10^{-3} as an ordinary number.

B2 · 3 marks · Meaning of a logarithm

(a) Write $\log_3 81 = 4$ in exponential form. (b) Write $5^{-2} = 1/25$ in logarithmic form. (c) Find x if $\log_x 64 = 3$.

B3 · 3 marks · Laws of logarithms

Write $2 \log 3 + \log 4 - \log 6$ as a single logarithm and simplify.

B4 · 3 marks · Laws of logarithms

Given log 2 = 0.3010 and log 3 = 0.4771, find (a) log 36 (b) log 1.5.

B5 · 3 marks · Characteristic and mantissa

Write the characteristic of (a) log 4520 (b) log 45.2 (c) log 0.0452.

B6 · 3 marks · Change of base

Use the change of base formula to evaluate $\log_8 32$.

Section C style: long questions (6 marks each)

C1 · 6 marks · Logarithmic equations

(a) Solve $\log_2(x + 3) + \log_2(x - 3) = 4$. (3 marks)

(b) Show that $\log_a b \times \log_b c = \log_a c$, and use it to find $\log_3 5 \times \log_5 81$. (3 marks)

C2 · 6 marks · Logarithms in real life

The loudness of a sound in decibels is $L = 10 \log(I / I_0)$, where I is the intensity in W/m^2 and $I_0 = 10^{-12}$ W/m^2.

(a) Find L for a busy street where $I = 10^{-5}$ W/m^2. (2 marks)

(b) A sound has $L = 90$ dB. Find its intensity. (2 marks)

(c) How many times more intense is a 90 dB sound than a 70 dB sound? (2 marks)

C3 · 6 marks · Logarithms in real life

A town has 5000 people, and its population grows by 4% each year, so after n years it is $5000 \times (1.04)^n$.

(a) Write an equation for the time when the population reaches 10 000. (2 marks)

(b) Take logarithms of both sides. (2 marks)

(c) Using log 2 = 0.3010 and log 1.04 = 0.0170, find how many whole years it takes to reach 10 000. (2 marks)

Model answers

B1.

1. (a) Move the decimal point 4 places to the right to get 7.03, so 0.000 703 = 7.03×10^{-4}.
2. (b) Move the decimal point 7 places to the left to get 3.84, so 38 400 000 = 3.84×10^{7}.
3. (c) A power of −3 moves the point 3 places to the left: $2.5 \times 10^{-3} = 0.0025$.

Answer: (a) 7.03×10^{-4} (b) 3.84×10^{7} (c) 0.0025

Watch out: The number in front must be at least 1 and less than 10, so 38.4×10^{6} is not in scientific notation.

B2.

1. $\log_b N = p$ means $b^p = N$.
2. (a) $3^4 = 81$
3. (b) $\log_5 (1/25) = -2$
4. (c) $x^3 = 64 = 4^3$, so $x = 4$.

Answer: (a) $3^4 = 81$ (b) $\log_5(1/25) = -2$ (c) $x = 4$

Watch out: The base of the logarithm becomes the base of the power: in $\log_3 81 = 4$, the base is 3, not 4.

B3.

1. Power law: $2 \log 3 = \log 3^2 = \log 9$.
2. Product law: $\log 9 + \log 4 = \log 36$.
3. Quotient law: $\log 36 - \log 6 = \log (36 \div 6) = \log 6$.

Answer: log 6

Watch out: Apply the power law first; 2 log 3 is log 9, not log 6.

B4.

1. (a) $36 = 2^2 \times 3^2$, so $\log 36 = 2 \log 2 + 2 \log 3 = 0.6020 + 0.9542 = 1.5562$.
2. (b) $1.5 = 3/2$, so $\log 1.5 = \log 3 - \log 2 = 0.4771 - 0.3010 = 0.1761$.

Answer: (a) 1.5562 (b) 0.1761

Watch out: Break the number into factors of 2 and 3 first, then use the product, quotient and power laws.

B5.

1. Write each number in scientific notation; the power of 10 is the characteristic.
2. (a) $4520 = 4.52 \times 10^{3}$, so the characteristic is 3.
3. (b) $45.2 = 4.52 \times 10^{1}$, so the characteristic is 1.
4. (c) $0.0452 = 4.52 \times 10^{-2}$, so the characteristic is −2 (written as 2 with a bar over it).
5. All three have the same mantissa, because the digits 452 are the same.

Answer: (a) 3 (b) 1 (c) −2

Watch out: For numbers less than 1, count the zeros after the decimal point and add 1: 0.0452 has one zero, so −2.

B6.

1. $\log_8 32 = \log_2 32 \div \log_2 8$
2. $\log_2 32 = 5$ because $2^5 = 32$, and $\log_2 8 = 3$ because $2^3 = 8$.
3. So $\log_8 32 = 5/3$.

Answer: 5/3

Watch out: Choose a new base that both numbers are powers of; here both 8 and 32 are powers of 2.

C1.

1. (a) Product law: $\log_2[(x + 3)(x - 3)] = 4$, so $(x + 3)(x - 3) = 2^4 = 16$.
2. $x^2 - 9 = 16$, so $x^2 = 25$ and $x = 5$ or $x = -5$.
3. Reject $x = -5$, because $\log_2(x - 3)$ needs $x - 3 > 0$. So $x = 5$.
4. (b) By change of base, $\log_a b = \log b / \log a$ and $\log_b c = \log c / \log b$.
5. Their product is $(\log b / \log a) \times (\log c / \log b) = \log c / \log a = \log_a c$.
6. So $\log_3 5 \times \log_5 81 = \log_3 81 = 4$, because $3^4 = 81$.

Answer: (a) $x = 5$ (b) 4

Watch out: Always check solutions of log equations: the number inside every logarithm must be positive.

C2.

1. (a) $I / I_0 = 10^{-5} \div 10^{-12} = 10^7$, so $L = 10 \log 10^7 = 10 \times 7 = 70$ dB.
2. (b) $90 = 10 \log(I / I_0)$, so $\log(I / I_0) = 9$ and $I / I_0 = 10^9$.
3. $I = 10^9 \times 10^{-12} = 10^{-3}$ W/m^2.
4. (c) The 70 dB sound has $I = 10^{-5}$. Ratio $= 10^{-3} \div 10^{-5} = 10^2 = 100$.

Answer: (a) 70 dB (b) 10^{-3} W/m^2 (c) 100 times

Watch out: An increase of 20 dB is not 20 times the intensity: every 10 dB multiplies the intensity by 10.

C3.

1. (a) $5000 \times (1.04)^n = 10\,000$, so $(1.04)^n = 2$.
2. (b) $\log (1.04)^n = \log 2$, so $n \log 1.04 = \log 2$.
3. (c) $n = 0.3010 \div 0.0170 \approx 17.7$.
4. After 17 years the population is still just below 10 000, so it first reaches 10 000 during the 18th year: 18 whole years.

Answer: (a) $(1.04)^n = 2$ (b) $n \log 1.04 = \log 2$ (c) about 17.7, so 18 years

Watch out: The power law $\log a^n = n \log a$ is what brings n down from the exponent.

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