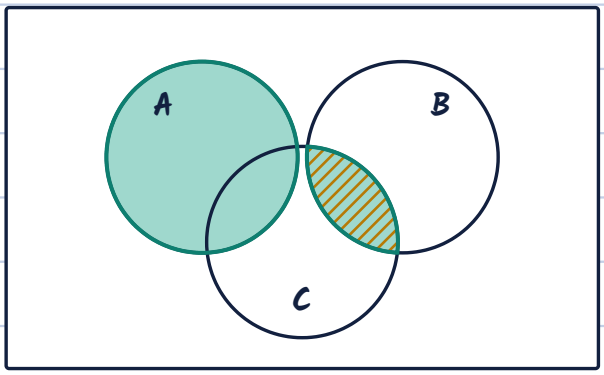
Exercise 3.1

Unit 3 · Sets and Relations

1 **Shade $A \cup (B \cap C)$, $A \cap (B \cup C)$, $(A \cup B) \cup C$ and $A \cap (B \cap C)$ using the following Venn diagrams.**

(i) *A and B do not overlap; C overlaps both.*

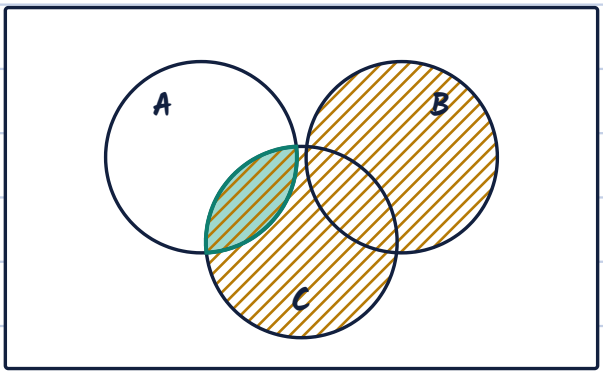
$A \cup (B \cap C)$



$B \cap C =$ ▨

$A \cup (B \cap C) =$ ▭

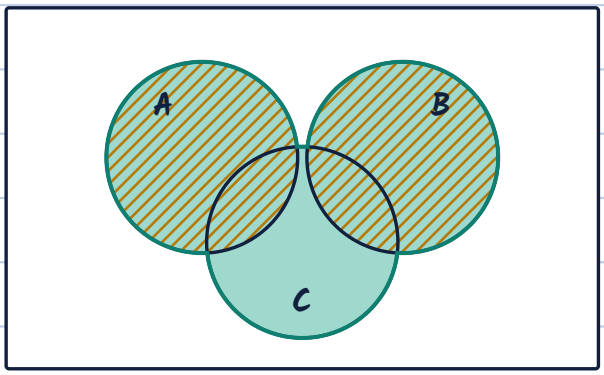
$A \cap (B \cup C)$



$B \cup C =$ ▨

$A \cap (B \cup C) =$ ▭

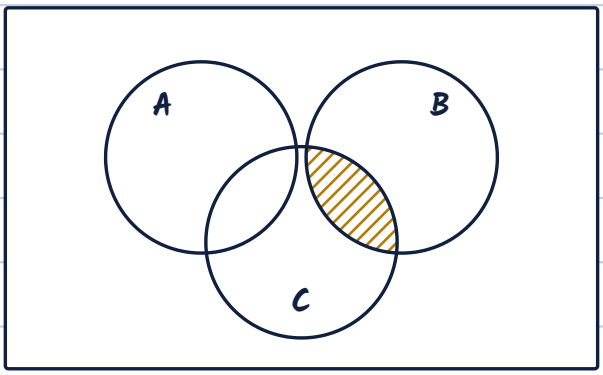
$(A \cup B) \cup C$



$A \cup B =$ ▨

$(A \cup B) \cup C =$ ▭

$A \cap (B \cap C)$

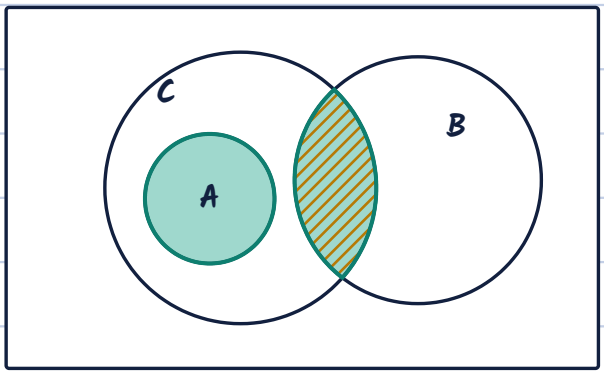


$B \cap C =$ ▨

$A \cap (B \cap C) = \{\}$ *(no region shaded)*

(ii) *A lies inside C; C overlaps B; A and B do not overlap.*

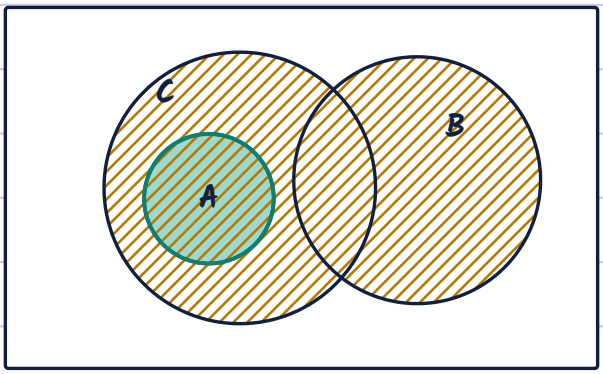
$A \cup (B \cap C)$



$B \cap C =$

$A \cup (B \cap C) =$

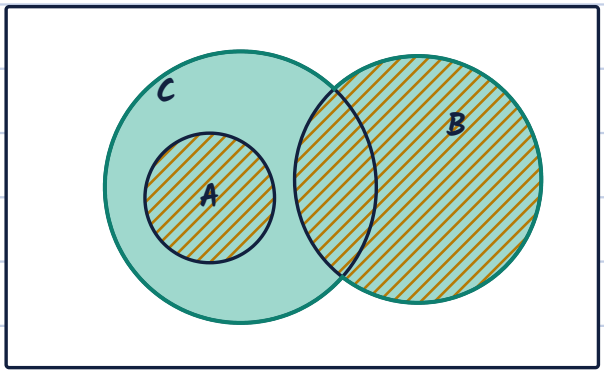
$A \cap (B \cup C)$



$B \cup C =$

$A \cap (B \cup C) =$

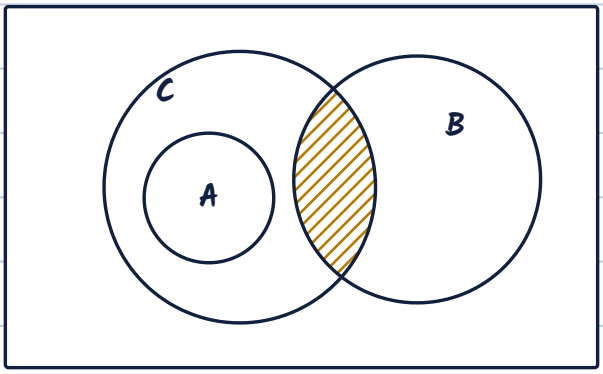
$(A \cup B) \cup C$



$A \cup B =$

$(A \cup B) \cup C =$

$A \cap (B \cap C)$

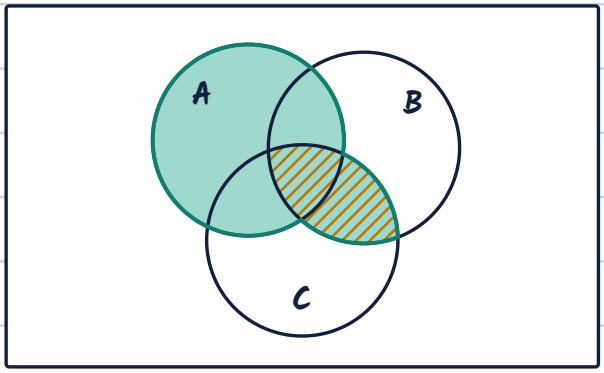


$B \cap C =$

$A \cap (B \cap C) = \{\ \}$ *(no region shaded)*

(iii) *A, B and C all overlap one another.*

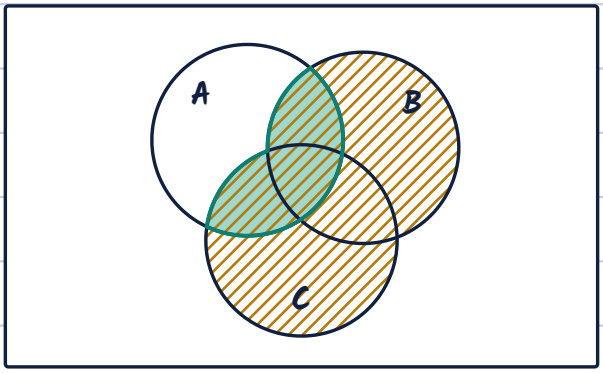
$A \cup (B \cap C)$



$B \cap C =$

$A \cup (B \cap C) =$

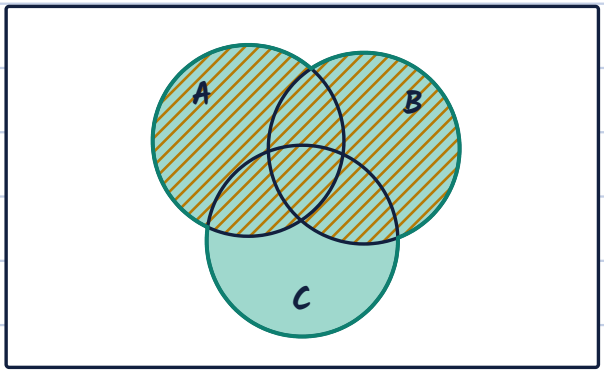
$A \cap (B \cup C)$



$B \cup C =$

$A \cap (B \cup C) =$

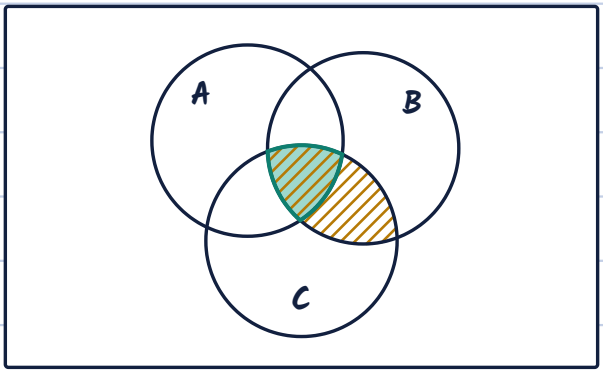
$(A \cup B) \cup C$



$A \cup B =$ ▨

$(A \cup B) \cup C =$ ▭

$A \cap (B \cap C)$

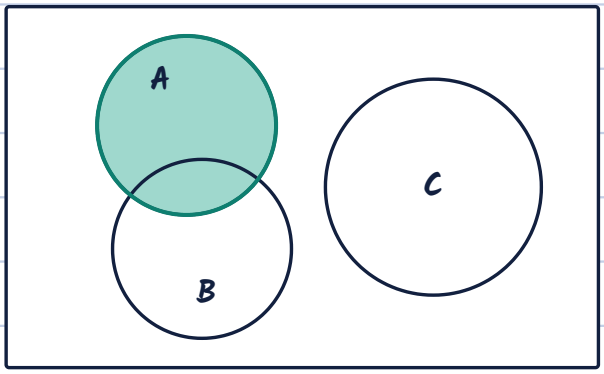


$B \cap C =$ ▨

$A \cap (B \cap C) =$ ▭

(iv) *A and B overlap; C does not overlap A or B.*

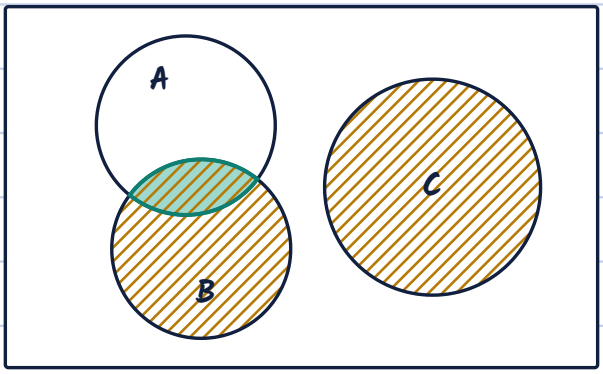
$A \cup (B \cap C)$



$B \cap C = \{\}$ *(no region shaded)*

$A \cup (B \cap C) =$ ▭

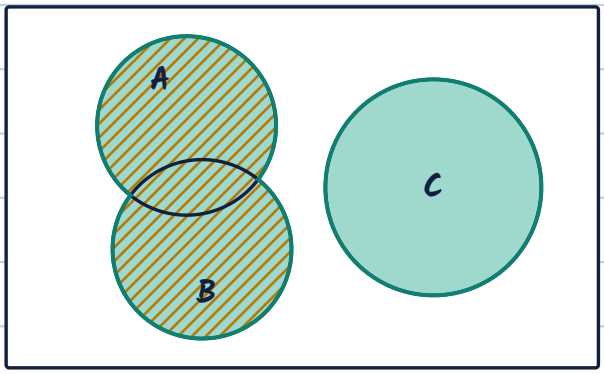
$A \cap (B \cup C)$



$B \cup C =$ ▨

$A \cap (B \cup C) =$ ▭

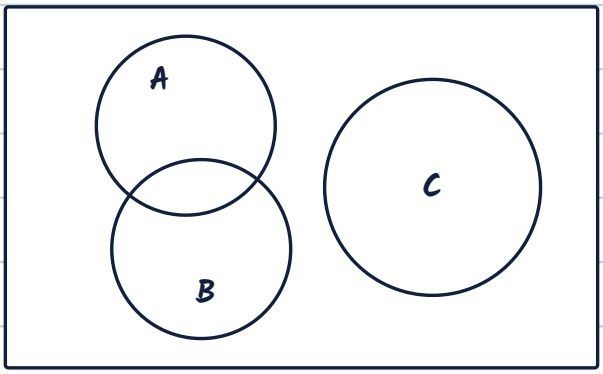
$(A \cup B) \cup C$



$A \cup B =$ ▨

$(A \cup B) \cup C =$ ▭

$A \cap (B \cap C)$



$B \cap C = \{\}$ *(no region shaded)*

$A \cap (B \cap C) = \{\}$ *(no region shaded)*

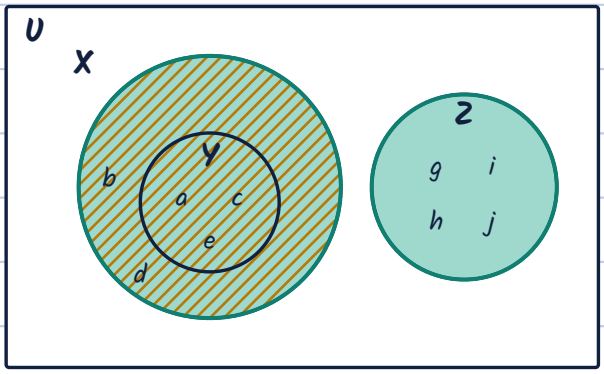
Key: the hatched region is the first step; the teal region (with a teal outline) is the final answer.

2 **If $X = \{a, b, c, d, e\}$, $Y = \{a, c, e\}$ and $Z = \{g, h, i, j\}$, then find the following using Venn diagrams.**

$X \cup Y = \{a, b, c, d, e\}$

$(X \cup Y) \cup Z = \{a, b, c, d, e, g, h, i, j\}$

(i) $(X \cup Y) \cup Z$



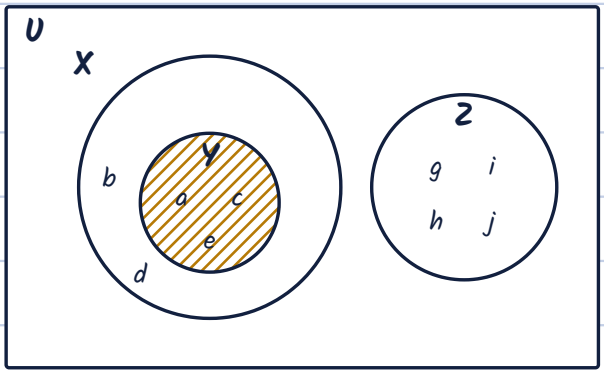
$X \cup Y =$ 

$(X \cup Y) \cup Z =$ 

$X \cap Y = \{a, c, e\}$

$(X \cap Y) \cap Z = \{\ \}$

(iii) $(X \cap Y) \cap Z$



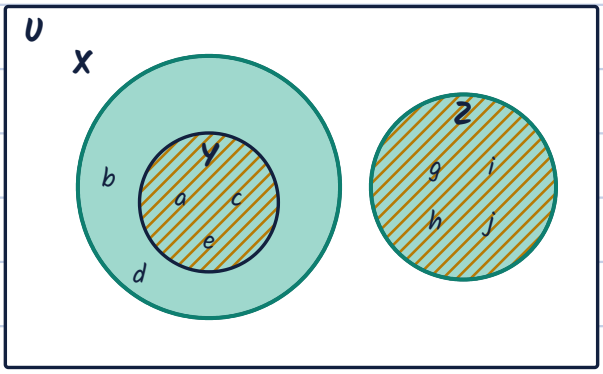
$X \cap Y =$ 

$(X \cap Y) \cap Z = \{\ \}$ *(no region shaded)*

$Y \cup Z = \{a, c, e, g, h, i, j\}$

$X \cup (Y \cup Z) = \{a, b, c, d, e, g, h, i, j\}$

(ii) $X \cup (Y \cup Z)$



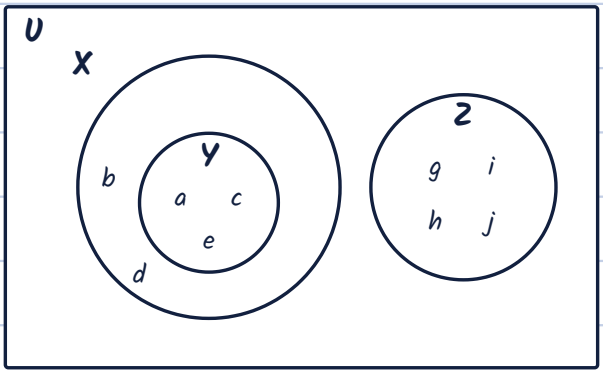
$Y \cup Z =$ 

$X \cup (Y \cup Z) =$ 

$Y \cap Z = \{\ \}$

$X \cap (Y \cap Z) = \{\ \}$

(iv) $X \cap (Y \cap Z)$



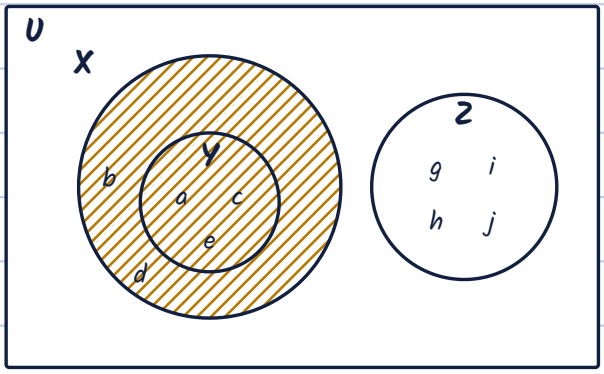
$Y \cap Z = \{\ \}$ *(no region shaded)*

$X \cap (Y \cap Z) = \{\ \}$ *(no region shaded)*

$X \cup Y = \{a, b, c, d, e\}$

$(X \cup Y) \cap Z = \{\ \}$

(v) $(X \cup Y) \cap Z$



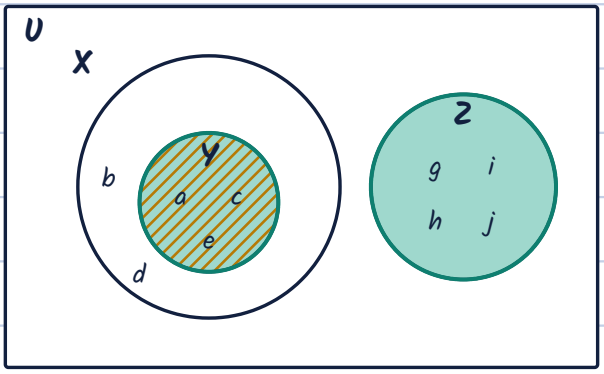
$X \cup Y =$ 

$(X \cup Y) \cap Z = \{\ \}$ *(no region shaded)*

$X \cap Y = \{a, c, e\}$

$(X \cap Y) \cup Z = \{a, c, e, g, h, i, j\}$

(vi) $(X \cap Y) \cup Z$



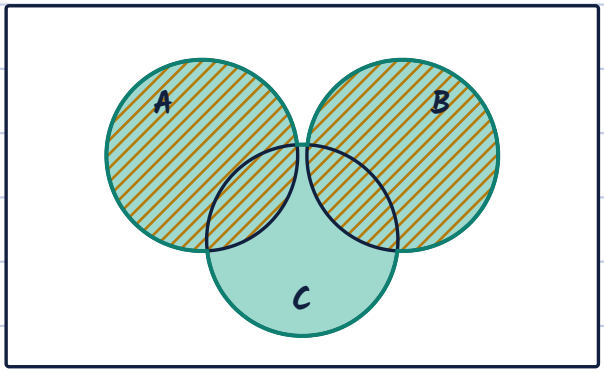
$X \cap Y =$ 

$(X \cap Y) \cup Z =$ 

3 Verify the associative laws of union and intersection by using the diagrams of Question 1.

(i) Union

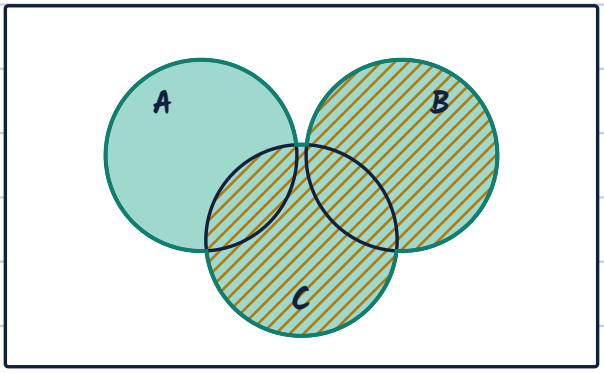
L.H.S = $(A \cup B) \cup C$



$A \cup B =$ 

$(A \cup B) \cup C =$ 

R.H.S = $A \cup (B \cup C)$



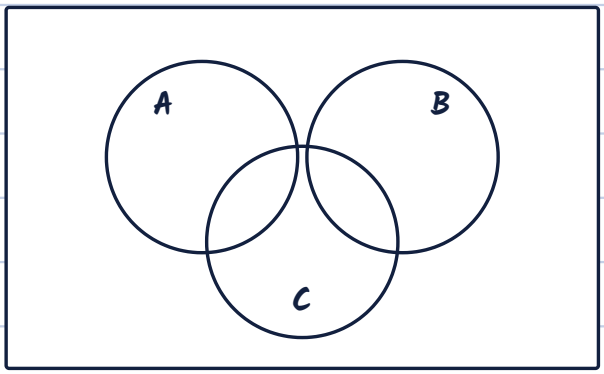
$B \cup C =$ 

$A \cup (B \cup C) =$ 

The same region is shaded in both diagrams. So, $(A \cup B) \cup C = A \cup (B \cup C)$. The associative law of union is verified.

Intersection

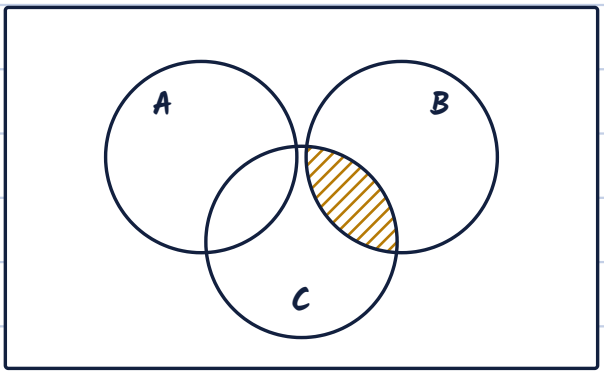
L.H.S = (A ∩ B) ∩ C



A ∩ B = { } (no region shaded)

(A ∩ B) ∩ C = { } (no region shaded)

R.H.S = A ∩ (B ∩ C)



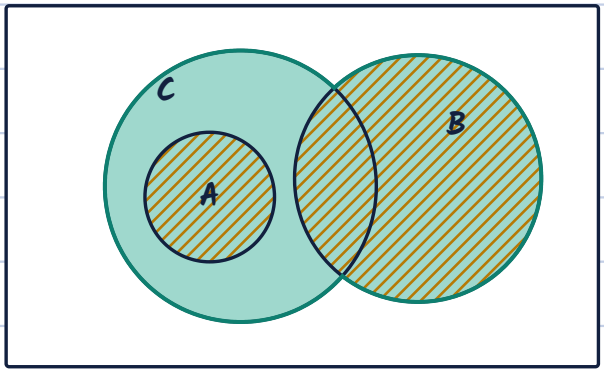
B ∩ C =

A ∩ (B ∩ C) = { } (no region shaded)

No region is shaded in either diagram: both sides are the empty set. So, (A ∩ B) ∩ C = A ∩ (B ∩ C). The associative law of intersection is verified.

(ii) Union

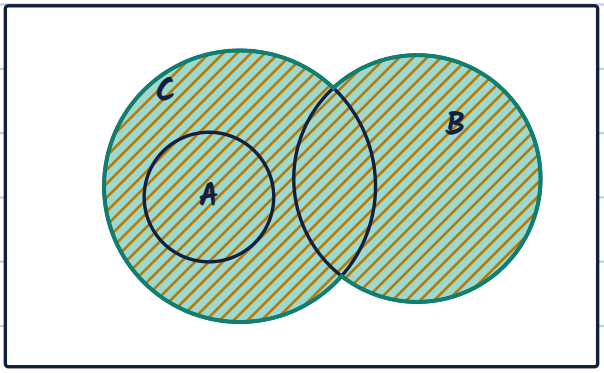
L.H.S = (A ∪ B) ∪ C



A ∪ B =

(A ∪ B) ∪ C =

R.H.S = A ∪ (B ∪ C)



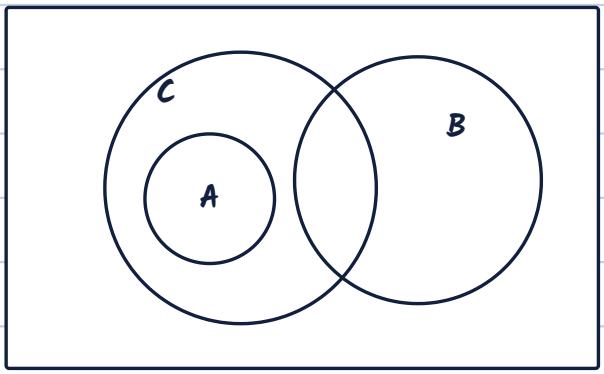
B ∪ C =

A ∪ (B ∪ C) =

The same region is shaded in both diagrams. So, (A ∪ B) ∪ C = A ∪ (B ∪ C). The associative law of union is verified.

Intersection

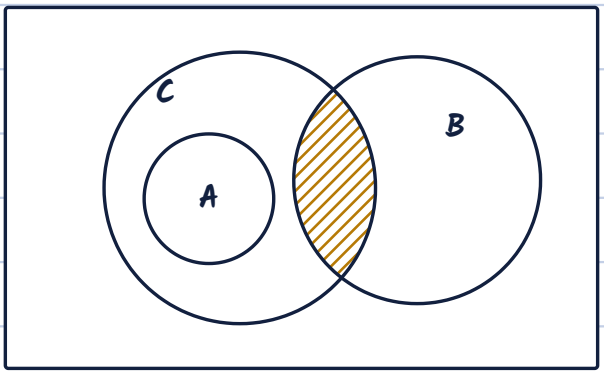
L.H.S = (A ∩ B) ∩ C



A ∩ B = { } (no region shaded)

(A ∩ B) ∩ C = { } (no region shaded)

R.H.S = A ∩ (B ∩ C)



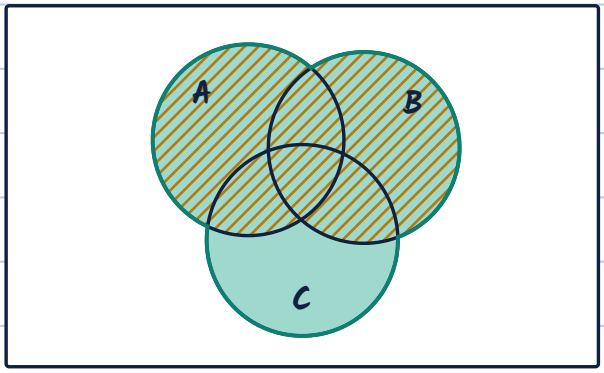
B ∩ C =

A ∩ (B ∩ C) = { } (no region shaded)

No region is shaded in either diagram: both sides are the empty set. So, (A ∩ B) ∩ C = A ∩ (B ∩ C). The associative law of intersection is verified.

(iii) Union

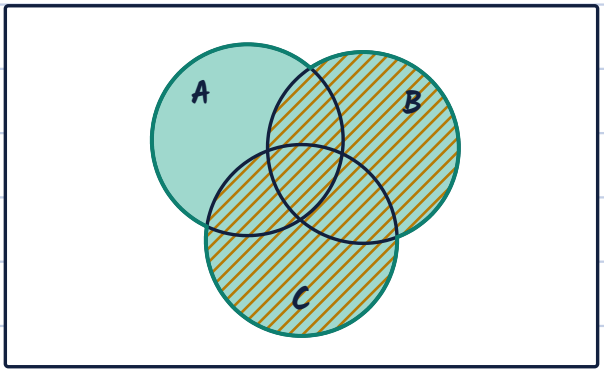
L.H.S = (A ∪ B) ∪ C



A ∪ B =

(A ∪ B) ∪ C =

R.H.S = A ∪ (B ∪ C)



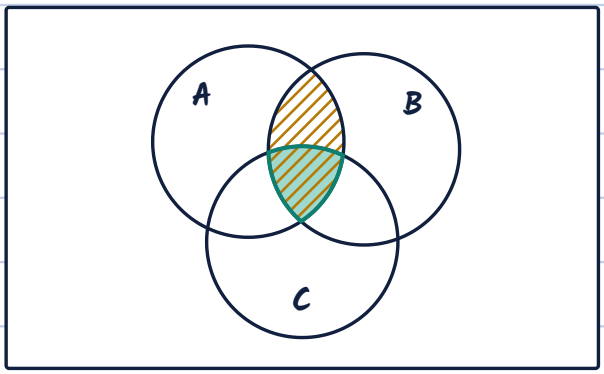
B ∪ C =

A ∪ (B ∪ C) =

The same region is shaded in both diagrams. So, (A ∪ B) ∪ C = A ∪ (B ∪ C). The associative law of union is verified.

Intersection

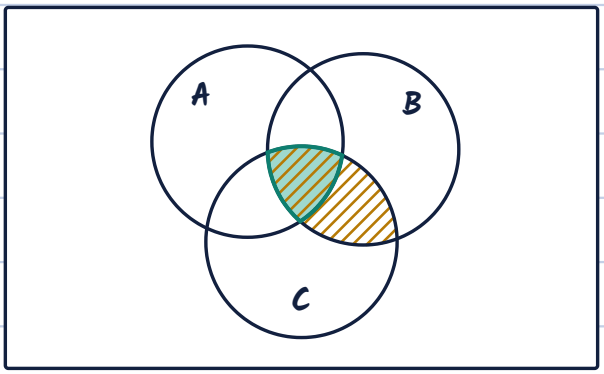
L.H.S = (A ∩ B) ∩ C



A ∩ B =

(A ∩ B) ∩ C =

R.H.S = A ∩ (B ∩ C)



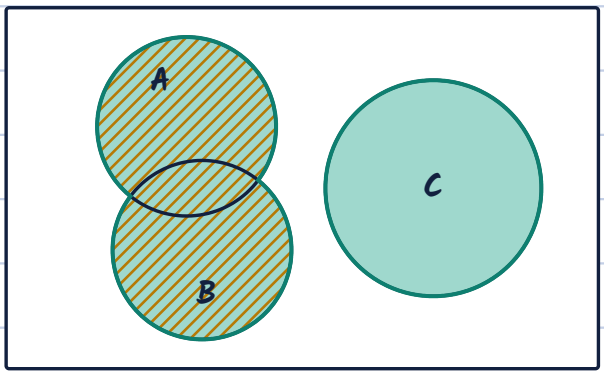
B ∩ C =

A ∩ (B ∩ C) =

The same region is shaded in both diagrams. So, (A ∩ B) ∩ C = A ∩ (B ∩ C). The associative law of intersection is verified.

(iv) Union

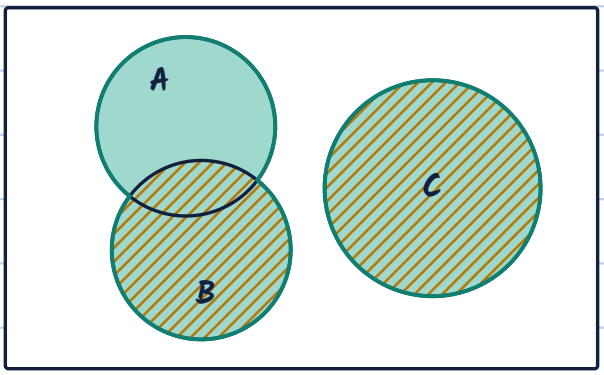
L.H.S = (A ∪ B) ∪ C



A ∪ B =

(A ∪ B) ∪ C =

R.H.S = A ∪ (B ∪ C)



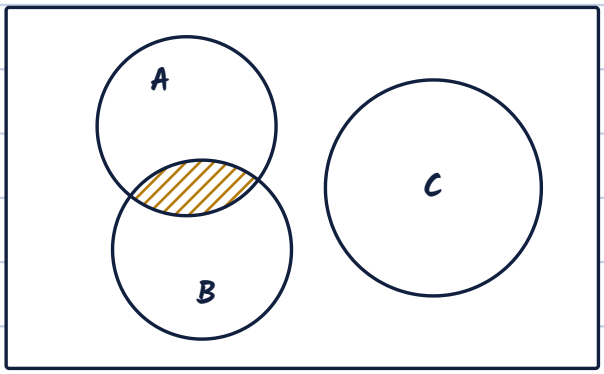
B ∪ C =

A ∪ (B ∪ C) =

The same region is shaded in both diagrams. So, (A ∪ B) ∪ C = A ∪ (B ∪ C). The associative law of union is verified.

Intersection

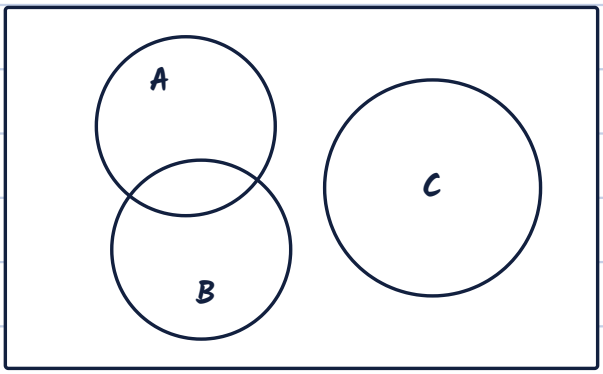
L.H.S = (A ∩ B) ∩ C



A ∩ B =

(A ∩ B) ∩ C = { } (no region shaded)

R.H.S = A ∩ (B ∩ C)



B ∩ C = { } (no region shaded)

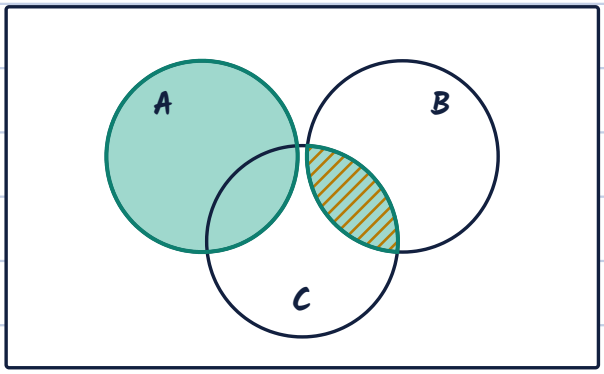
A ∩ (B ∩ C) = { } (no region shaded)

No region is shaded in either diagram: both sides are the empty set. So, (A ∩ B) ∩ C = A ∩ (B ∩ C). The associative law of intersection is verified.

4 Verify the distributive laws by using the diagrams of Question 1.

(i) (a) Union over intersection

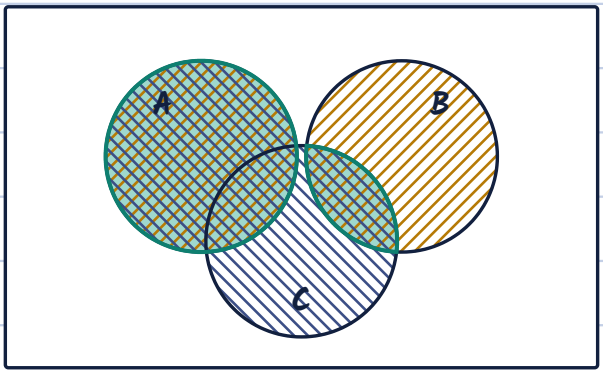
L.H.S = A ∪ (B ∩ C)



B ∩ C =

A ∪ (B ∩ C) =

R.H.S = (A ∪ B) ∩ (A ∪ C)



A ∪ B =

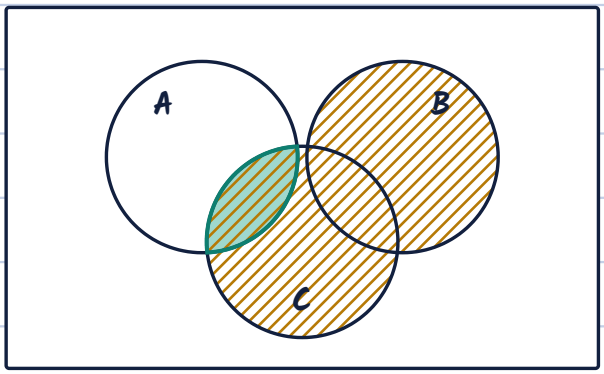
A ∪ C =

(A ∪ B) ∩ (A ∪ C) =

The same region is shaded in both diagrams. So, A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C). The distributive law of union over intersection is verified.

(b) Intersection over union

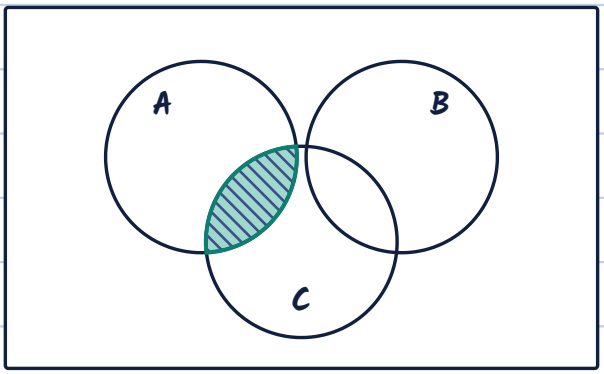
L.H.S = A ∩ (B ∪ C)



B ∪ C =

A ∩ (B ∪ C) =

R.H.S = (A ∩ B) ∪ (A ∩ C)



A ∩ B = { } (no region shaded)

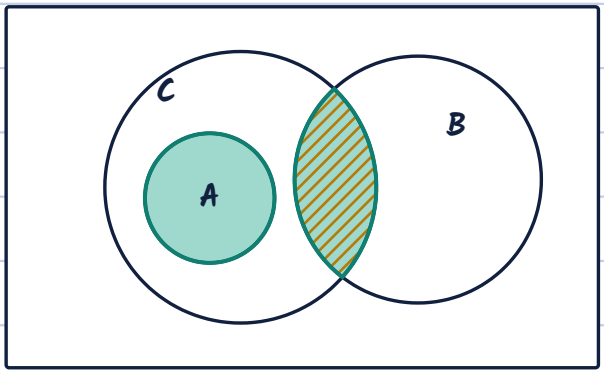
A ∩ C =

(A ∩ B) ∪ (A ∩ C) =

The same region is shaded in both diagrams. So, A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C). The distributive law of intersection over union is verified.

(ii) (a) Union over intersection

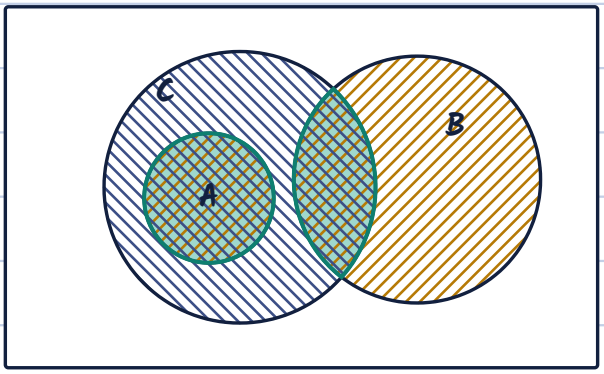
L.H.S = A ∪ (B ∩ C)



B ∩ C =

A ∪ (B ∩ C) =

R.H.S = (A ∪ B) ∩ (A ∪ C)



A ∪ B =

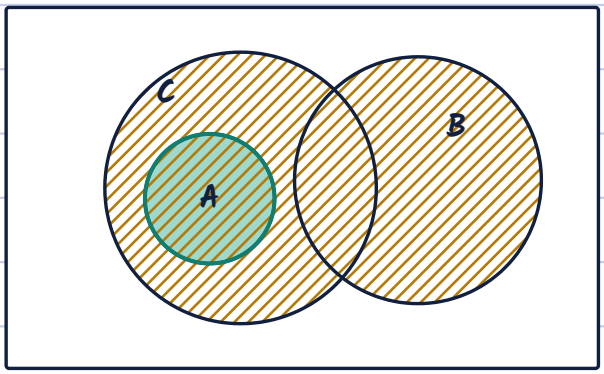
A ∪ C =

(A ∪ B) ∩ (A ∪ C) =

The same region is shaded in both diagrams. So, A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C). The distributive law of union over intersection is verified.

(b) Intersection over union

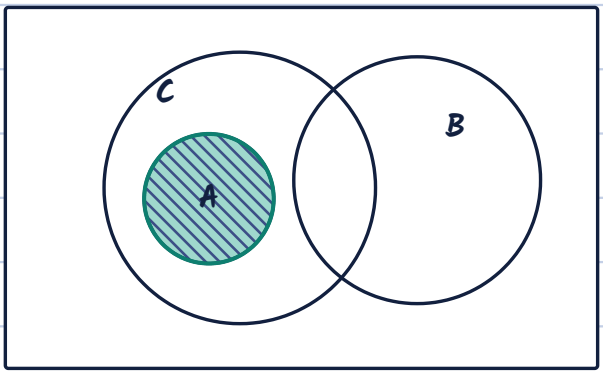
L.H.S = A ∩ (B ∪ C)



B ∪ C =

A ∩ (B ∪ C) =

R.H.S = (A ∩ B) ∪ (A ∩ C)



A ∩ B = { } (no region shaded)

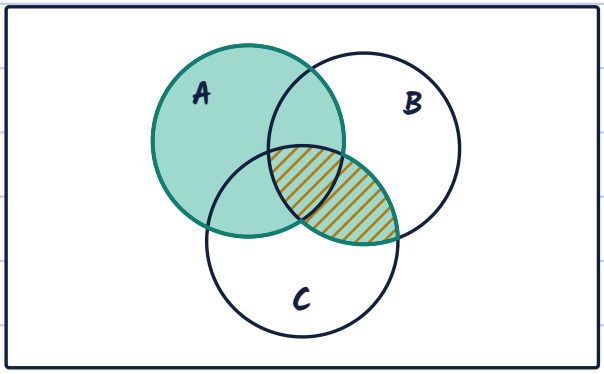
A ∩ C =

(A ∩ B) ∪ (A ∩ C) =

The same region is shaded in both diagrams. So, A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C). The distributive law of intersection over union is verified.

(iii) (a) Union over intersection

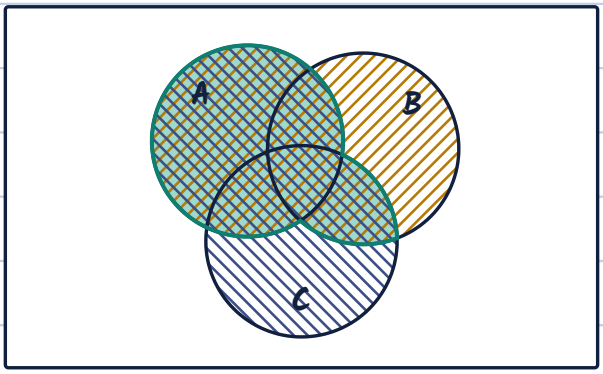
L.H.S = A ∪ (B ∩ C)



B ∩ C =

A ∪ (B ∩ C) =

R.H.S = (A ∪ B) ∩ (A ∪ C)



A ∪ B =

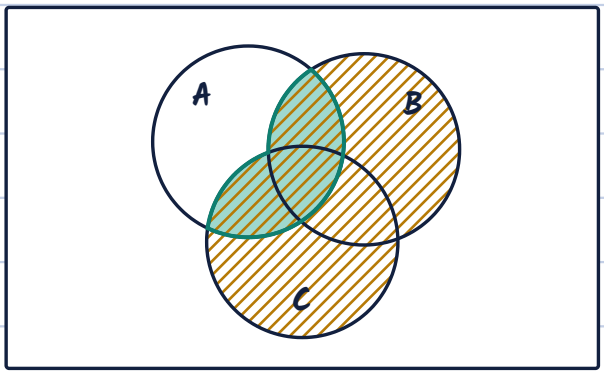
A ∪ C =

(A ∪ B) ∩ (A ∪ C) =

The same region is shaded in both diagrams. So, A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C). The distributive law of union over intersection is verified.

(b) Intersection over union

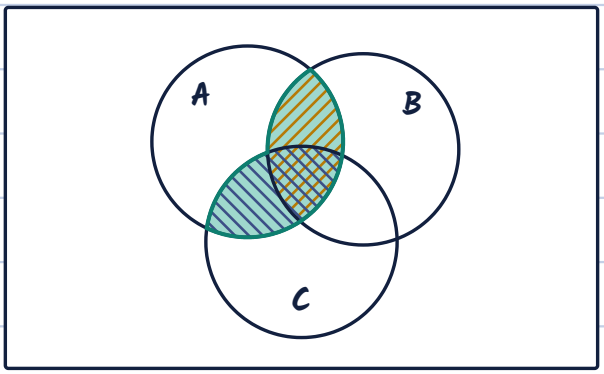
L.H.S = A ∩ (B ∪ C)



B ∪ C =

A ∩ (B ∪ C) =

R.H.S = (A ∩ B) ∪ (A ∩ C)



A ∩ B =

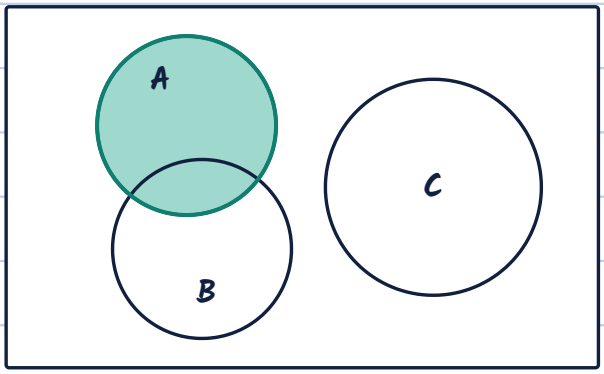
A ∩ C =

(A ∩ B) ∪ (A ∩ C) =

The same region is shaded in both diagrams. So, A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C). The distributive law of intersection over union is verified.

(iv) (a) Union over intersection

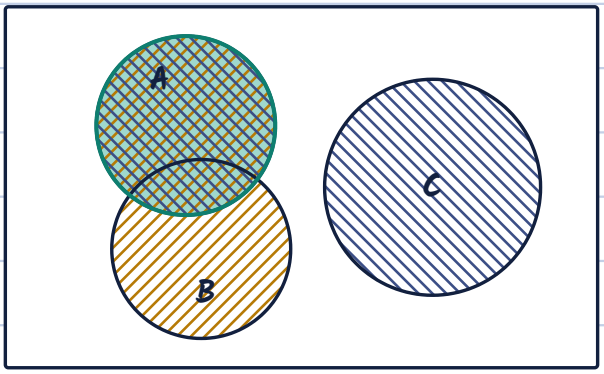
L.H.S = A ∪ (B ∩ C)



B ∩ C = { } (no region shaded)

A ∪ (B ∩ C) =

R.H.S = (A ∪ B) ∩ (A ∪ C)



A ∪ B =

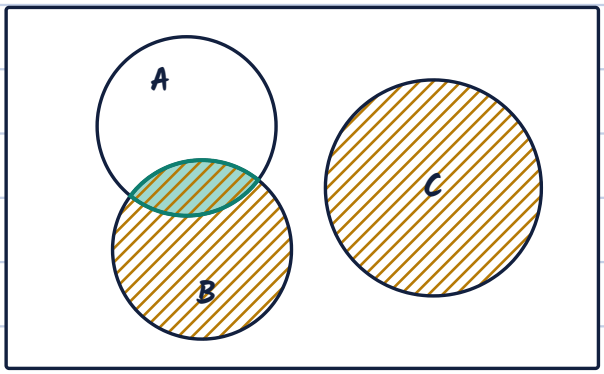
A ∪ C =

(A ∪ B) ∩ (A ∪ C) =

The same region is shaded in both diagrams. So, A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C). The distributive law of union over intersection is verified.

(b) Intersection over union

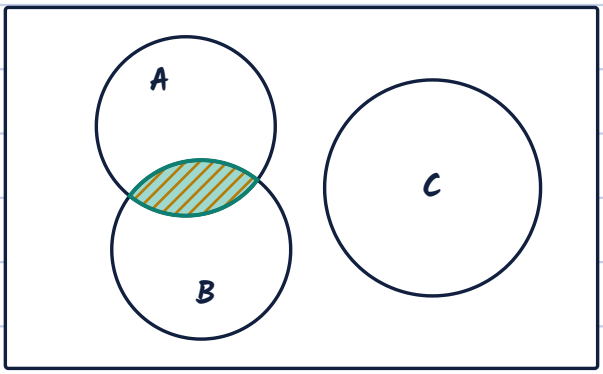
L.H.S = A ∩ (B ∪ C)



B ∪ C =

A ∩ (B ∪ C) =

R.H.S = (A ∩ B) ∪ (A ∩ C)



A ∩ B =

A ∩ C = { } (no region shaded)

(A ∩ B) ∪ (A ∩ C) =

The same region is shaded in both diagrams. So, A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C). The distributive law of intersection over union is verified.

5 Prove by using Venn diagrams.

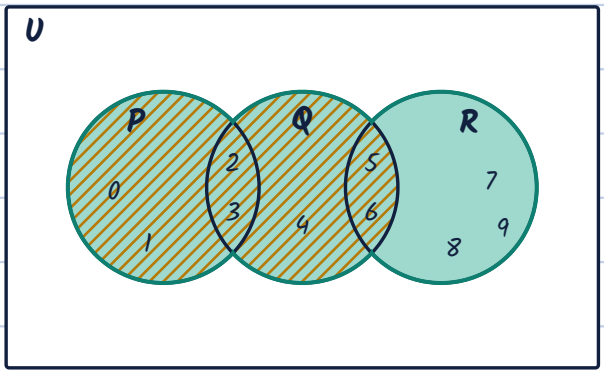
(i) P = {0, 1, 2, 3}, Q = {2, 3, 4, 5, 6}, R = {5, 6, 7, 8, 9}

(a) (P ∪ Q) ∪ R = P ∪ (Q ∪ R)

P ∪ Q = {0, 1, 2, 3, 4, 5, 6}

(P ∪ Q) ∪ R = {0, 1, 2, ..., 9}

L.H.S = (P ∪ Q) ∪ R



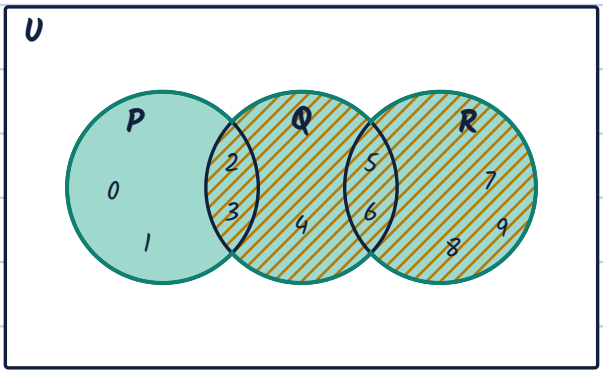
P ∪ Q =

(P ∪ Q) ∪ R =

Q ∪ R = {2, 3, 4, 5, 6, 7, 8, 9}

P ∪ (Q ∪ R) = {0, 1, 2, ..., 9}

R.H.S = P ∪ (Q ∪ R)



Q ∪ R =

P ∪ (Q ∪ R) =

The same region is shaded in both diagrams. Hence, (P ∪ Q) ∪ R = P ∪ (Q ∪ R).

(b) $(P \cap Q) \cap R = P \cap (Q \cap R)$

$P \cap Q = \{2, 3\}$

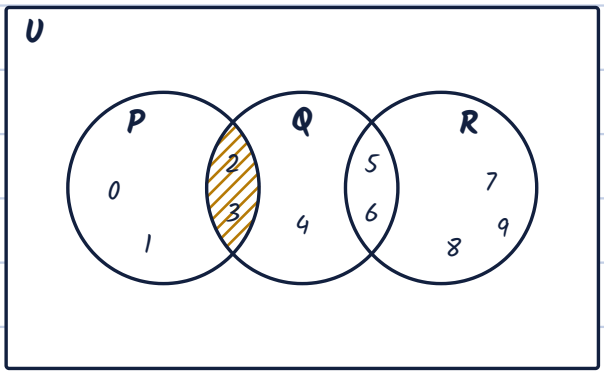
$(P \cap Q) \cap R = \{\ \}$

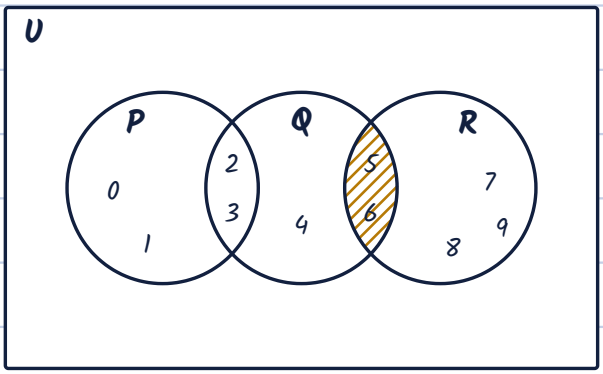
L.H.S = $(P \cap Q) \cap R$

$Q \cap R = \{5, 6\}$

$P \cap (Q \cap R) = \{\ \}$

R.H.S = $P \cap (Q \cap R)$





$P \cap Q =$ ▨

$(P \cap Q) \cap R = \{\ \}$ *(no region shaded)*

$Q \cap R =$ ▨

$P \cap (Q \cap R) = \{\ \}$ *(no region shaded)*

No region is shaded in either diagram: both sides are the empty set. Hence, $(P \cap Q) \cap R = P \cap (Q \cap R)$.

(ii) $P = \{m, n, o, p, q\},\ Q = \{r, s, t, u\},\ R = \{t, u, v, w\}$

(a) $(P \cup Q) \cup R = P \cup (Q \cup R)$

$P \cup Q = \{m, n, o, p, q, r, s, t, u\}$

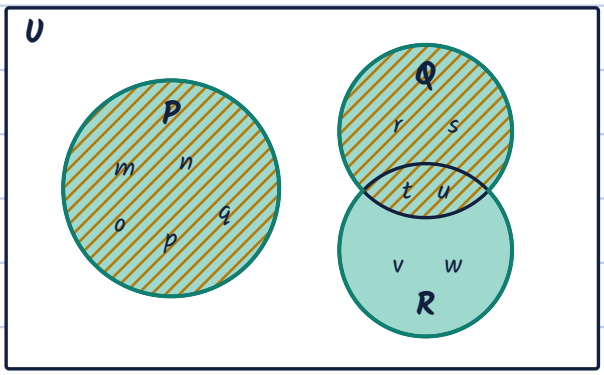
$(P \cup Q) \cup R = \{m, n, o, \ldots, w\}$

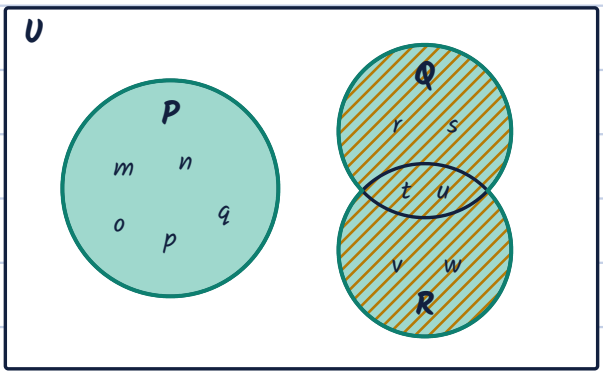
L.H.S = $(P \cup Q) \cup R$

$Q \cup R = \{r, s, t, u, v, w\}$

$P \cup (Q \cup R) = \{m, n, o, \ldots, w\}$

R.H.S = $P \cup (Q \cup R)$





$P \cup Q =$ ▨

$(P \cup Q) \cup R =$ ▭

$Q \cup R =$ ▨

$P \cup (Q \cup R) =$ ▭

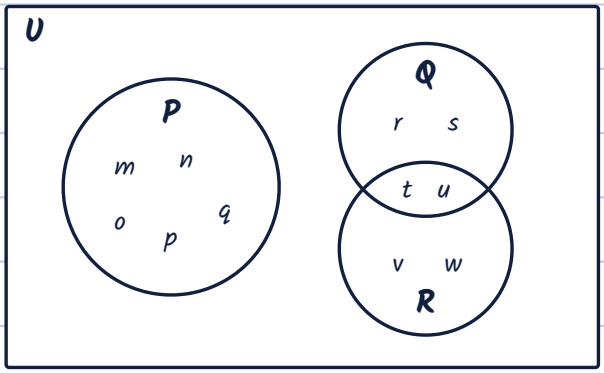
The same region is shaded in both diagrams. Hence, $(P \cup Q) \cup R = P \cup (Q \cup R)$.

(b) $(P \cap Q) \cap R = P \cap (Q \cap R)$

$P \cap Q = \{\ \}$

$(P \cap Q) \cap R = \{\ \}$

L.H.S = $(P \cap Q) \cap R$



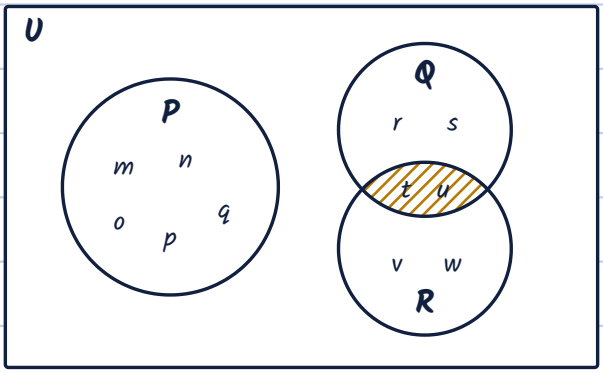
$P \cap Q = \{\ \}$ (no region shaded)

$(P \cap Q) \cap R = \{\ \}$ (no region shaded)

$Q \cap R = \{t, u\}$

$P \cap (Q \cap R) = \{\ \}$

R.H.S = $P \cap (Q \cap R)$



$Q \cap R =$ 

$P \cap (Q \cap R) = \{\ \}$ (no region shaded)

No region is shaded in either diagram: both sides are the empty set. Hence, $(P \cap Q) \cap R = P \cap (Q \cap R)$.

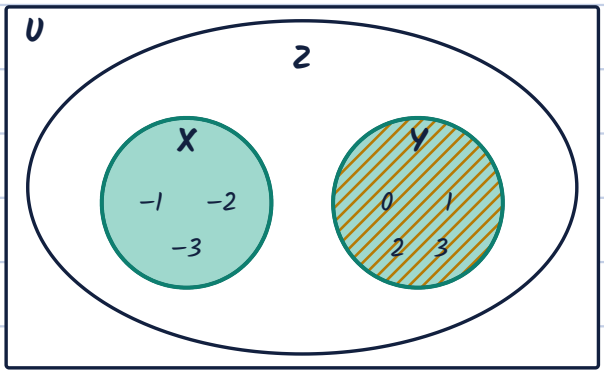
6 **Verify $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$ using Venn diagrams for the following sets.**

$X = \{-1, -2, -3\},\ Y = \{0, 1, 2, 3\},\ Z = \{0, \pm1, \pm2, \pm3\}$

$Y \cap Z = \{0, 1, 2, 3\} = Y$

$X \cup (Y \cap Z) = \{0, \pm1, \pm2, \pm3\}$

L.H.S = $X \cup (Y \cap Z)$



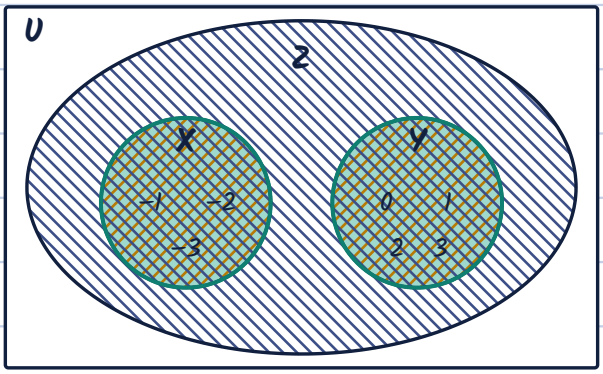
$Y \cap Z =$ 

$X \cup (Y \cap Z) =$ 

$X \cup Y = \{0, \pm1, \pm2, \pm3\},\ X \cup Z = Z$

$(X \cup Y) \cap (X \cup Z) = \{0, \pm1, \pm2, \pm3\}$

R.H.S = $(X \cup Y) \cap (X \cup Z)$



$X \cup Y =$ 

$X \cup Z =$ 

$(X \cup Y) \cap (X \cup Z) =$ 

The same region is shaded in both diagrams. So, $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$. (Both sides equal $\{0, \pm1, \pm2, \pm3\}$, which is the set Z.)

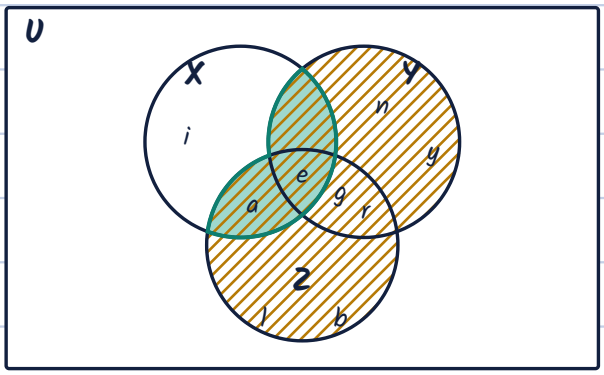
7 **Verify $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$, where X = the set of the first three vowels, Y = the set of letters of the word "energy" and Z = the set of letters of the word "algebra".**

$X = \{a, e, i\}$, $Y = \{e, n, r, g, y\}$, $Z = \{a, l, g, e, b, r\}$

$Y \cup Z = \{a, e, g, l, b, r, y, n\}$

$X \cap (Y \cup Z) = \{a, e\}$

L.H.S = $X \cap (Y \cup Z)$



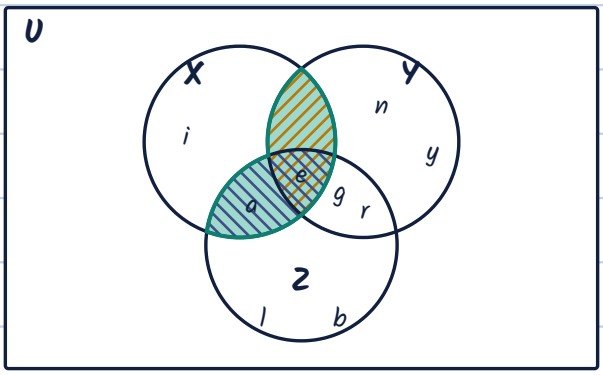
$Y \cup Z =$ ▨

$X \cap (Y \cup Z) =$ ▭

$X \cap Y = \{e\}$, $X \cap Z = \{a, e\}$

$(X \cap Y) \cup (X \cap Z) = \{a, e\}$

R.H.S = $(X \cap Y) \cup (X \cap Z)$



$X \cap Y =$ ▨

$X \cap Z =$ ▨

$(X \cap Y) \cup (X \cap Z) =$ ▭

The same region is shaded in both diagrams. So, $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$.