Exercise 3.2

Unit 3 · Sets and Relations

1 **Let A and B be two finite sets such that $n(A) = 24$, $n(B) = 18$ and $n(A \cup B) = 31$. Find $n(A \cap B)$.**

For any two overlapping sets A and B:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

so, $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 24 + 18 - 31 = 11$

$\therefore n(A \cap B) = 11$

2 **Given $n(A - B) = 23$, $n(A \cup B) = 44$ and $n(A \cap B) = 2$, find $n(B)$ and $n(B - A)$.**

$n(A - B) = n(A \cup B) - n(B)$

so, $n(B) = n(A \cup B) - n(A - B) = 44 - 23 = 21$

Also, $n(B - A) = n(B) - n(A \cap B) = 21 - 2 = 19$

$\therefore n(B) = 21$ and $n(B - A) = 19$

3 **Students who like algebra (A) and geometry (G): find the number who like geometry and draw a Venn diagram.**

Like algebra or geometry, $n(A \cup G) = 30$

Like algebra, $n(A) = 20$

Like both, $n(A \cap G) = 15$

(ii) Number of students who like geometry

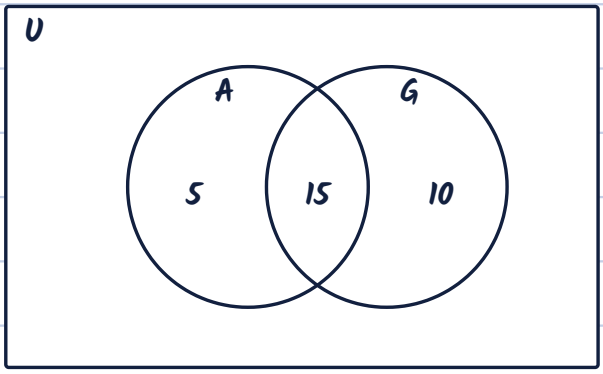
$n(A \cup G) = n(A) + n(G) - n(A \cap G)$

$n(G) = n(A \cup G) - n(A) + n(A \cap G) = 30 - 20 + 15 = 25$

$\therefore$ 25 students like geometry.

(i) Venn diagram

Algebra only = 20 − 15 = 5, both = 15, geometry only = 25 − 15 = 10



4 Houses with a lawn (L) and a car porch (P): how many houses have neither?

Total houses, $n(U)$ = 50

Houses with a lawn, $n(L)$ = 25

Houses with a car porch, $n(P)$ = 32

Houses with both, $n(L \cap P)$ = 15

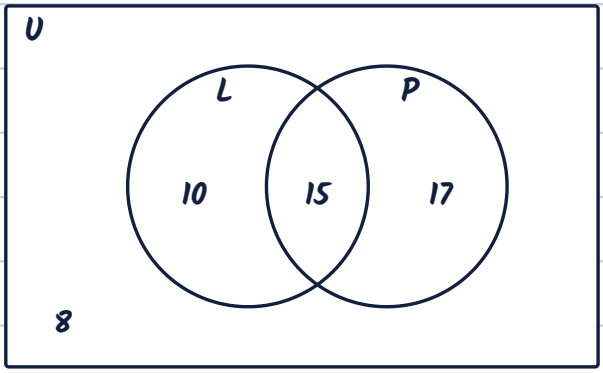
$n(L \cup P) = n(L) + n(P) - n(L \cap P) = 25 + 32 - 15 = 42$

Houses with neither = $n(L \cup P)' = n(U) - n(L \cup P)$

$= 50 - 42 = 8$

∴ 8 houses have neither a lawn nor a car porch.

Lawn only = 25 − 15 = 10, both = 15, car porch only = 32 − 15 = 17, neither = 8



5 Children at primary (P), elementary (E) and secondary (S) level: how many children are out of school?

Total number of children, $n(U)$ = 940

Primary level, $n(P)$ = 400

Elementary level, $n(E)$ = 240

Secondary level, $n(S)$ = 175

A child studies at only one level, so P, E and S are disjoint sets.

For disjoint sets:

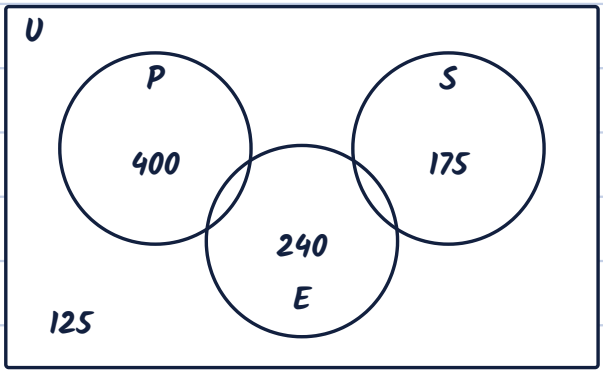
$n(P \cup E \cup S) = n(P) + n(E) + n(S)$

$= 400 + 240 + 175 = 815$

Out of school $= n(P \cup E \cup S)' = n(U) - n(P \cup E \cup S)$

$= 940 - 815 = 125$

∴ 125 children are out of school.



6 **Customers who like mango (M), vanilla (V) and chocolate (C) flavours.**

Like mango, $n(M)$ = 100

Like vanilla, $n(V)$ = 90

Like chocolate, $n(C)$ = 40

Mango and vanilla, $n(M \cap V)$ = 20

Chocolate and vanilla, $n(C \cap V)$ = 14

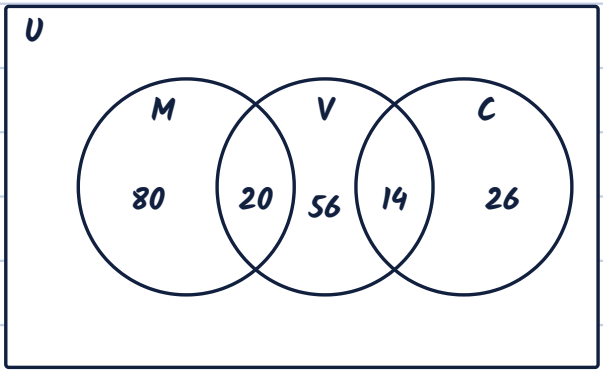
Only mango $= n(M) - n(M \cap V) = 100 - 20 = 80$

Only vanilla $= n(V) - n(M \cap V) - n(C \cap V)$

$= 90 - 20 - 14 = 56$

Only chocolate $= n(C) - n(C \cap V) = 40 - 14 = 26$

∴ Only mango 80, only vanilla 56, only chocolate 26



7 **Students with a laptop (L), a cell phone (C) and an iPod (I).**

$n(U) = 200$ (Students interviewed)
$n(L) = 42$ (Laptop)
$n(C) = 80$ (Cell phone)
$n(I) = 100$ (iPod)
$n(L \cap C) = 23$
$n(L \cap I) = 10$
$n(C \cap I) = 14$
$n(L \cap C \cap I) = 8$

(a) Students who have only a cell phone

$= n(C) - n(L \cap C) - n(C \cap I) + n(L \cap C \cap I)$
$= 80 - 23 - 14 + 8 = 51$

∴ 51 students have only a cell phone.

(b) Students who have none of the three items

$n(L \cup C \cup I) = n(L) + n(C) + n(I) - n(L \cap C)$
$- n(L \cap I) - n(C \cap I) + n(L \cap C \cap I)$
$= 42 + 80 + 100 - 23 - 10 - 14 + 8 = 183$
None $= n(U) - n(L \cup C \cup I) = 200 - 183 = 17$

∴ 17 students have none of these items.

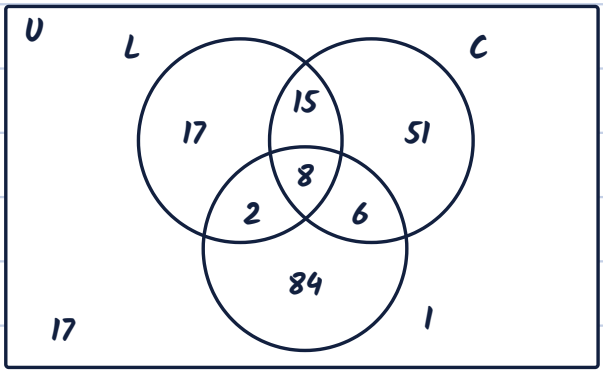
(c) Students who have an iPod and a laptop but not a cell phone

$= n(L \cap I) - n(L \cap C \cap I) = 10 - 8 = 2$

∴ 2 students have an iPod and a laptop but not a cell phone.

If the question means an iPod or a laptop (or both) but no cell phone, add the three regions outside C: $17 + 2 + 84 = 103$.

Regions: laptop only = 42 − 15 − 2 − 8 = 17, L and C only = 23 − 8 = 15, L and I only = 10 − 8 = 2, C and I only = 14 − 8 = 6, iPod only = 100 − 2 − 6 − 8 = 84, none = 17



8 **Students who play badminton (B) and tennis (T): find the total number of students.**

Play badminton, $n(B)$ = 350

Play tennis, $n(T)$ = 280

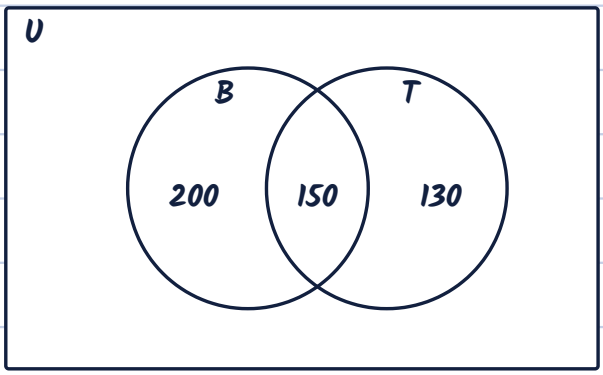
Play both, $n(B \cap T)$ = 150

$n(B \cup T) = n(B) + n(T) - n(B \cap T)$

$= 350 + 280 - 150 = 480$

∴ There are 480 students in the college.

Badminton only = 350 − 150 = 200, both = 150, tennis only = 280 − 150 = 130



9 **Students learning English (E) and Chinese (C): how many are learning Chinese?**

Total students, $n(E \cup C)$ = 50

Learning English, $n(E)$ = 26

Learning both, $n(E \cap C)$ = 8

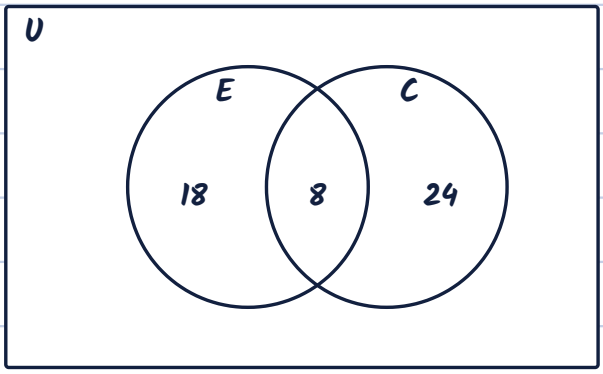
$n(E \cup C) = n(E) + n(C) - n(E \cap C)$

$50 = 26 + n(C) - 8$

$n(C) = 50 - 26 + 8 = 32$

∴ 32 students are learning Chinese.

English only = $26 - 8 = 18$, both = 8, Chinese only = $32 - 8 = 24$



10 People who like tea (T) and coffee (C): how many like both?

Total people, $n(T \cup C)$ = 70

Like tea, $n(T)$ = 48

Like coffee, $n(C)$ = 40

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$n(T \cap C) = n(T) + n(C) - n(T \cup C) = 48 + 40 - 70 = 18$

∴ 18 people like both tea and coffee.

Tea only = $48 - 18 = 30$, both = 18, coffee only = $40 - 18 = 22$

