Exercise 4.1

Factorize the following polynomials.

1) $2x^2y^3 - 6x^2y^2 + 2xy^3$

$\therefore 2xy^2(xy - 3x + y)$

2) $3nx - 3x - 3ny + 3y$

$= 3x(n - 1) - 3y(n - 1)$

$= (n - 1)(3x - 3y)$

$\therefore 3(n - 1)(x - y)$

3) $18x^4 + 108x^2y^2 + 162y^4$

$= 18(x^4 + 6x^2y^2 + 9y^4)$

$= 18(x^4 + 3x^2y^2 + 3x^2y^2 + 9y^4)$

$= 18[x^2(x^2 + 3y^2) + 3y^2(x^2 + 3y^2)]$

$= 18(x^2 + 3y^2)(x^2 + 3y^2)$

$\therefore 18(x^2 + 3y^2)^2$

4) $(k + 2)^2 - 8(k + 2) + 16$

$= (k + 2)^2 - 4(k + 2) - 4(k + 2) + 16$

$= (k + 2)(k + 2 - 4) - 4(k + 2 - 4)$

$= (k + 2)(k - 2) - 4(k - 2)$

$= (k - 2)(k + 2 - 4) = (k - 2)(k - 2)$

$\therefore (k - 2)^2$

5) $9x^2 + 4 - 169y^2 - 12x$

$= 9x^2 - 12x + 4 - 169y^2$

$= (3x)^2 - 2(3x)(2) + (2)^2 - (13y)^2$

$= (3x - 2)^2 - (13y)^2$

$\therefore (3x - 2 + 13y)(3x - 2 - 13y)$

6) $(x^2 - 1)(y + 1) - (y + 3)(x^2 - 1)$

$= (x^2 - 1)(y + 1 - y - 3)$

$= (x^2 - 1)(-2)$

$\therefore -2(x + 1)(x - 1)$

7) $x^2 - 6ax + 9a^2 - 16b^2$

$= (x)^2 - 2(x)(3a) + (3a)^2 - (4b)^2$

$= (x - 3a)^2 - (4b)^2$

$\therefore (x - 3a - 4b)(x - 3a + 4b)$

8) $1 - x^2 - 2xy - y^2$

$= 1 - (x^2 + 2xy + y^2)$

$= (1)^2 - (x + y)^2$

$\therefore (1 - x - y)(1 + x + y)$

9 **Find a polynomial whose factorization is $(x + y - 2c)(x + 2c + y)$, by using an appropriate formula.**

$(x + y - 2c)(x + 2c + y) = (x + y - 2c)(x + y + 2c)$

$= (x + y)^2 - (2c)^2$ [using $(a - b)(a + b) = a^2 - b^2$]

$\therefore x^2 + 2xy + y^2 - 4c^2$

10 **Show the expression $x^2 + 4y^2 - z^2 + 4xy$ as the difference of two squares.**

$x^2 + 4y^2 - z^2 + 4xy = x^2 + 4xy + 4y^2 - z^2$

$= (x)^2 + 2(x)(2y) + (2y)^2 - z^2$

$\therefore (x + 2y)^2 - z^2$

11 **Find the missing factor in the following.**

(a) $2y^2 - 3y - 27 = (y + 3)(\quad)$

$2y^2 - 3y - 27 = 2y^2 - 9y + 6y - 27$

$= y(2y - 9) + 3(2y - 9) = (2y - 9)(y + 3)$

$\therefore$ Missing factor $= 2y - 9$

(b) $5x^2 + 12x - 9 = (\quad)(x + 3)$

$5x^2 + 12x - 9 = 5x^2 + 15x - 3x - 9$

$= 5x(x + 3) - 3(x + 3) = (x + 3)(5x - 3)$

$\therefore$ Missing factor $= 5x - 3$

Factorize the following.

12) $x^4 + 4m^4$

$= (x^2)^2 + (2m^2)^2 + 2(x^2)(2m^2) - 2(x^2)(2m^2)$

$= (x^2 + 2m^2)^2 - 4x^2m^2$

$= (x^2 + 2m^2)^2 - (2xm)^2$

$\therefore (x^2 + 2m^2 - 2xm)(x^2 + 2m^2 + 2xm)$

13) $m^4 + m^2 + 1$

$= (m^2)^2 + (1)^2 + 2(m^2)(1) - 2(m^2)(1) + m^2$

$= (m^2 + 1)^2 - 2m^2 + m^2$

$= (m^2 + 1)^2 - (m)^2$

$\therefore (m^2 + 1 - m)(m^2 + 1 + m)$

14) $-3x^4 - 21x^3 + 24x^2$

$= -3x^2(x^2 + 7x - 8)$

$= -3x^2(x^2 + 8x - x - 8)$

$= -3x^2[x(x + 8) - 1(x + 8)]$

$\therefore -3x^2(x + 8)(x - 1)$

15) $x^8 + x^4 + 1$

$= (x^4)^2 + (1)^2 + 2(x^4)(1) - 2(x^4)(1) + x^4$

$= (x^4 + 1)^2 - 2x^4 + x^4 = (x^4 + 1)^2 - (x^2)^2$

$= (x^4 + 1 - x^2)(x^4 + 1 + x^2)$

$= (x^4 - x^2 + 1)(x^4 + 2x^2 + 1 - x^2)$

$= (x^4 - x^2 + 1)[(x^2 + 1)^2 - (x)^2]$

$\therefore (x^4 - x^2 + 1)(x^2 - x + 1)(x^2 + x + 1)$

16) $4x^4 + 256y^4$

$= (2x^2)^2 + (16y^2)^2 + 2(2x^2)(16y^2) - 2(2x^2)(16y^2)$

$= (2x^2 + 16y^2)^2 - 64x^2y^2 = (2x^2 + 16y^2)^2 - (8xy)^2$

$= (2x^2 + 16y^2 + 8xy)(2x^2 + 16y^2 - 8xy)$

$= 2(x^2 + 8y^2 + 4xy) \times 2(x^2 + 8y^2 - 4xy)$

$\therefore 4(x^2 + 4xy + 8y^2)(x^2 - 4xy + 8y^2)$

17) $12 - 7x + x^2$

$= x^2 - 7x + 12$

$= (x)^2 - 2(x)(\frac{7}{2}) + (\frac{7}{2})^2 - (\frac{7}{2})^2 + 12$

$= (x - \frac{7}{2})^2 - \frac{49}{4} + 12 = (x - \frac{7}{2})^2 - \frac{1}{4}$

$= (x - \frac{7}{2})^2 - (\frac{1}{2})^2$

$= (x - \frac{7}{2} + \frac{1}{2})(x - \frac{7}{2} - \frac{1}{2})$

$\therefore (x - 3)(x - 4)$

Another way: $x^2 - 4x - 3x + 12 = x(x - 4) - 3(x - 4) = (x - 4)(x - 3)$

18) $x^2 - 9x + 8$

$= x^2 - 8x - x + 8$

$= x(x - 8) - 1(x - 8)$

$\therefore (x - 8)(x - 1)$

19) $10z^2 - 29z + 10$

$= 10z^2 - 25z - 4z + 10$

$= 5z(2z - 5) - 2(2z - 5)$

$\therefore (2z - 5)(5z - 2)$

20) $-3y^2 + 13y - 4$

$= -(3y^2 - 13y + 4)$

$= -(3y^2 - 12y - y + 4)$

$= -[3y(y - 4) - 1(y - 4)]$

$= -(y - 4)(3y - 1)$

$\therefore (4 - y)(3y - 1)$

21) $x^2 - 21x + 90$

$= x^2 - 15x - 6x + 90$

$= x(x - 15) - 6(x - 15)$

$\therefore (x - 15)(x - 6)$

22) $x^2 + x - 2$

$= x^2 + 2x - x - 2$

$= x(x + 2) - 1(x + 2)$

$\therefore (x + 2)(x - 1)$

23) $3x^2 + 11x + 6$

$= 3x^2 + 9x + 2x + 6$

$= 3x(x + 3) + 2(x + 3)$

$\therefore (x + 3)(3x + 2)$

24) $2x^2 - 5xy - 3y^2$

$= 2x^2 - 6xy + xy - 3y^2$

$= 2x(x - 3y) + y(x - 3y)$

$\therefore (x - 3y)(2x + y)$

25) $8 + 6x - 5x^2$

$= -(5x^2 - 6x - 8)$

$= -(5x^2 - 10x + 4x - 8)$

$= -[5x(x - 2) + 4(x - 2)]$

$= -(x - 2)(5x + 4)$

$\therefore (2 - x)(5x + 4)$

26) $6 - 7x - 5x^2$

$= 6 - 10x + 3x - 5x^2$

$= 2(3 - 5x) + x(3 - 5x)$

$\therefore (3 - 5x)(x + 2)$

27) $2a^2 - 4a - 6$

$= 2(a^2 - 2a - 3)$

$= 2(a^2 - 3a + a - 3)$

$= 2[a(a - 3) + 1(a - 3)]$

$\therefore 2(a - 3)(a + 1)$

28) $u^4 - 13u^2 + 36$

$= u^4 - 9u^2 - 4u^2 + 36$

$= u^2(u^2 - 9) - 4(u^2 - 9)$

$= (u^2 - 9)(u^2 - 4)$

$\therefore (u + 3)(u - 3)(u + 2)(u - 2)$

29) $y^4 - 12y^2 - 64$

$= y^4 - 16y^2 + 4y^2 - 64$

$= y^2(y^2 - 16) + 4(y^2 - 16)$

$= (y^2 - 16)(y^2 + 4)$

$\therefore (y + 4)(y - 4)(y^2 + 4)$