Exercise 4.3

Unit 4 · Factorization and Algebraic Manipulation

1 (a) Find the HCF of the following monomials by completing the table.

Monomials	HCF of numbers	p	q	r	HCF
$16p^3q,\ 9pq^2r$	1	p	q	—	pq
$10p^3q^2r,\ 5p^2qr,\ 15p^2qr^2$	5	p^2	q	r	$5p^2qr$
$14p^4qr^4,\ 28p^3qr^2,\ 7p^2qr^2,\ 21p^2q^2r^4$	7	p^2	q	r^2	$7p^2qr^2$

How the numbers were found:

$16 = 2 \times 2 \times 2 \times 2,\ 9 = 3 \times 3 \Rightarrow$ HCF = 1

$10 = 2 \times 5,\ 5 = 1 \times 5,\ 15 = 3 \times 5 \Rightarrow$ HCF = 5

$14 = 2 \times 7,\ 28 = 4 \times 7,\ 7 = 1 \times 7,\ 21 = 3 \times 7 \Rightarrow$ HCF = 7

For each variable, take the lowest power that appears in every term. If a variable is missing from any term, it is not part of the HCF (r in the first row).

(b) If all the common factors, with least power, of three unknown polynomials are 2^2, 3, pq and $(p + q)^2$, what would be their HCF?

$HCF = 2^2 \times 3 \times pq \times (p + q)^2$

$\therefore HCF = 12pq(p + q)^2$

(c) Write any two polynomials of your choice having HCF as 1.

$(x + y)^2$ and $(x - y)^2$ have no common factor.

$\therefore$ HCF of $(x + y)^2$ and $(x - y)^2 = 1$

(d) Two polynomials have only one common factor $m - n$ and only one uncommon factor $m^2 + n^2$. Find the polynomials.

Common factor $= m - n$, uncommon factor $= m^2 + n^2$

$\therefore$ The polynomials are $(m - n)$ and $(m - n)(m^2 + n^2)$

(e) Can you guess the HCF of $x^3 + 5x + 1$ and $1 + 5x + x^3$ without any procedure?

Both are the same polynomial written in a different order.

$\therefore$ HCF $= x^3 + 5x + 1$

Find the HCF of the following by factorization.

2) $(x + y)^2$, $x^2 - y^2$

$(x + y)^2 = (x + y)(x + y)$

$x^2 - y^2 = (x + y)(x - y)$

Common factors $= x + y$

$\therefore$ Required HCF $= x + y$

3) $(a - b)^3$, $a^2 - 2ab + b^2$

$(a - b)^3 = (a - b)(a - b)(a - b)$

$a^2 - 2ab + b^2 = (a - b)(a - b)$

Common factors $= (a - b), (a - b)$

$\therefore$ Required HCF $= (a - b)^2 = a^2 - 2ab + b^2$

4) $a^3b - ab^3$, $a^5b^2 - a^2b^5$

$a^3b - ab^3 = ab(a^2 - b^2) = ab(a + b)(a - b)$

$a^5b^2 - a^2b^5 = a^2b^2(a^3 - b^3) = a^2b^2(a - b)(a^2 + ab + b^2)$

Common factors $= a, b, (a - b)$ (with least power)

$\therefore$ Required HCF $= ab(a - b)$

5) $x^2 - 49$, $x^2 - 4x - 21$

$x^2 - 49 = (x + 7)(x - 7)$

$x^2 - 4x - 21 = x^2 - 7x + 3x - 21$

$= x(x - 7) + 3(x - 7) = (x - 7)(x + 3)$

Common factors $= x - 7$

$\therefore$ Required HCF $= x - 7$

6) $12x^2 + x - 1,\ 15x^2 + 8x + 1$

$12x^2 + x - 1 = 12x^2 + 4x - 3x - 1 = 4x(3x + 1) - 1(3x + 1)$

$= (3x + 1)(4x - 1)$

$15x^2 + 8x + 1 = 15x^2 + 5x + 3x + 1 = 5x(3x + 1) + 1(3x + 1)$

$= (3x + 1)(5x + 1)$

Common factors $= 3x + 1$

$\therefore$ Required HCF $= 3x + 1$

7) $c^2x^2 - d^2,\ acx^2 - bcx + adx - bd$

$c^2x^2 - d^2 = (cx)^2 - (d)^2 = (cx + d)(cx - d)$

$acx^2 - bcx + adx - bd = cx(ax - b) + d(ax - b)$

$= (ax - b)(cx + d)$

Common factors $= cx + d$

$\therefore$ Required HCF $= cx + d$

8) $m^2 - n^2,\ m^4 - n^4,\ m^6 - n^6$

$m^2 - n^2 = (m + n)(m - n)$

$m^4 - n^4 = (m^2 + n^2)(m^2 - n^2) = (m^2 + n^2)(m + n)(m - n)$

$m^6 - n^6 = (m^2)^3 - (n^2)^3 = (m^2 - n^2)(m^4 + m^2n^2 + n^4)$

$= (m + n)(m - n)(m^4 + m^2n^2 + n^4)$

Common factors $= (m + n), (m - n)$

$\therefore$ Required HCF $= (m + n)(m - n) = m^2 - n^2$

9) $ax^2 + 2a^2x + a^3,\ 2ax^2 - 4a^2x - 6a^3,\ 3(ax + a^2)^2$

$ax^2 + 2a^2x + a^3 = a(x^2 + 2ax + a^2)$

$= a(x + a)^2 = a(x + a)(x + a)$

$2ax^2 - 4a^2x - 6a^3 = 2a(x^2 - 2ax - 3a^2)$

$= 2a(x^2 - 3ax + ax - 3a^2)$

$= 2a[x(x - 3a) + a(x - 3a)] = 2a(x - 3a)(x + a)$

$3(ax + a^2)^2 = 3[a(x + a)]^2 = 3a^2(x + a)(x + a)$

Common factors $= a, (x + a)$

$\therefore$ Required HCF $= a(x + a)$