Exercise 4.4

Unit 4 · Factorization and Algebraic Manipulation

1 **The HCF of the two polynomials $x^3 + 5x^2 + 6x$ and $x^3 + 9x^2 + 14x$ is $x^2 + 2x$. Find the HCF in each case.**

(i) HCF of $5(x^3 + 5x^2 + 6x)$ and $x^3 + 9x^2 + 14x = x^2 + 2x$

5 is not a factor of the second polynomial, so the HCF does not change.

(ii) HCF of $x^3 + 5x^2 + 6x$ and $2(x^3 + 9x^2 + 14x) = x^2 + 2x$

(iii) HCF of $3(x^3 + 5x^2 + 6x)$ and $7(x^3 + 9x^2 + 14x) = x^2 + 2x$

3 and 7 have no common factor, so the HCF does not change.

(iv) HCF of $15(x^3 + 5x^2 + 6x)$ and $25(x^3 + 9x^2 + 14x)$
$= 5(x^2 + 2x)$

The HCF of 15 and 25 is 5, so the HCF is multiplied by 5.

(v) If both polynomials are multiplied by x, then
HCF $= x(x^2 + 2x) = x^3 + 2x^2$

2 **Find the LCM of the following monomials.**

Monomials	LCM of numbers	x	y	z	LCM
$8x^6y$, $4x^2yz$	8	x^6	y	z	$8x^6yz$
$12x^2y^4z$, $24x^3z$	24	x^3	y^4	z	$24x^3y^4z$
$18x^3z$, $9xy^2z$, $6x^6yz^3$	18	x^6	y^2	z^3	$18x^6y^2z^3$
$14xyz^3$, z^5y^3x, $28x^3y^5z$	28	x^3	y^5	z^5	$28x^3y^5z^5$

For each variable, take the highest power that appears in any of the monomials.

Find the HCF of the following by the division method.

3) $a^2 + a - 2,\ a^3 + 2a^2 + a + 2$

$$
\begin{array}{r|l}
a^2 + a - 2 & a^3 + 2a^2 + a + 2 \\
 & -(a^3 + a^2 - 2a) \\
\hline
 & a^2 + 3a + 2 \\
 & -(a^2 + a - 2) \\
\hline
 & 2a + 4
\end{array}
$$

(quotient: $a + 1$)

$= 2(a + 2)$

Leave out the factor 2 and divide $a^2 + a - 2$ by $a + 2$:

$$
\begin{array}{r|l}
a + 2 & a^2 + a - 2 \\
 & -(a^2 + 2a) \\
\hline
 & -a - 2 \\
 & -(-a - 2) \\
\hline
 & 0
\end{array}
$$

(quotient: $a - 1$)

∴ Required HCF = $a + 2$

4) $x^3 + 2x^2 - 4x - 8,\ 2x^3 + 7x^2 + 4x - 4$

$$
\begin{array}{r|l}
x^3 + 2x^2 - 4x - 8 & 2x^3 + 7x^2 + 4x - 4 \\
 & -(2x^3 + 4x^2 - 8x - 16) \\
\hline
 & 3x^2 + 12x + 12
\end{array}
$$

(quotient: 2)

$= 3(x^2 + 4x + 4)$

Leave out the factor 3 and divide by $x^2 + 4x + 4$:

$$
\begin{array}{r|l}
x^2 + 4x + 4 & x^3 + 2x^2 - 4x - 8 \\
 & -(x^3 + 4x^2 + 4x) \\
\hline
 & -2x^2 - 8x - 8 \\
 & -(-2x^2 - 8x - 8) \\
\hline
 & 0
\end{array}
$$

(quotient: $x - 2$)

∴ Required HCF = $x^2 + 4x + 4 = (x + 2)^2$

5) $2x^3 + x^2 - x - 2,\ 3x^3 - x^2 + x - 3$

Multiply the dividend by 2 so that the first term divides exactly.

$$\begin{array}{r|l} & 3 \\ 2x^3 + x^2 - x - 2 & 3x^3 - x^2 + x - 3 \\ & \times\ 2 \\ & 6x^3 - 2x^2 + 2x - 6 \\ & -(6x^3 + 3x^2 - 3x - 6) \\ & \hline & -5x^2 + 5x \end{array}$$

$$= -5x(x - 1)$$

Leave out the factor $-5x$ and divide by $x - 1$:

$$\begin{array}{r|l} & 2x^2 + 3x + 2 \\ x - 1 & 2x^3 + x^2 - x - 2 \\ & -(2x^3 - 2x^2) \\ & \hline & 3x^2 - x - 2 \\ & -(3x^2 - 3x) \\ & \hline & 2x - 2 \\ & -(2x - 2) \\ & \hline & 0 \end{array}$$

$\therefore$ *Required HCF* $= x - 1$

6) $3 + 2p^4 + 5p^2,\ 5p + 5p^3 + 3 + 3p^2$

First arrange both in descending powers of p:

$2p^4 + 5p^2 + 3$ *and* $5p^3 + 3p^2 + 5p + 3$

Multiply the dividend by 5:

$$\begin{array}{r|l} & 2p \\ 5p^3 + 3p^2 + 5p + 3 & 2p^4 + 5p^2 + 3 \\ & \times\ 5 \\ & 10p^4 + 25p^2 + 15 \\ & -(10p^4 + 6p^3 + 10p^2 + 6p) \\ & \hline & -6p^3 + 15p^2 - 6p + 15 \end{array}$$

$$= -3(2p^3 - 5p^2 + 2p - 5)$$

Leave out -3. Multiply $5p^3 + 3p^2 + 5p + 3$ by 2 and divide by $2p^3 - 5p^2 + 2p - 5$:

$$\begin{array}{r|l} & 5 \\ 2p^3 - 5p^2 + 2p - 5 & 5p^3 + 3p^2 + 5p + 3 \\ & \times 2 \\ & 10p^3 + 6p^2 + 10p + 6 \\ & -(10p^3 - 25p^2 + 10p - 25) \\ & \hline & 31p^2 + 31 \end{array}$$

$$= 31(p^2 + 1)$$

Leave out 31 and divide by $p^2 + 1$:

$$\begin{array}{r|l} & 2p - 5 \\ p^2 + 1 & 2p^3 - 5p^2 + 2p - 5 \\ & -(2p^3 + 2p) \\ & \hline & -5p^2 - 5 \\ & -(-5p^2 - 5) \\ & \hline & 0 \end{array}$$

$\therefore$ Required HCF $= p^2 + 1$

7) $24x^4 - 2x^3 - 60x^2 - 32x,\ 18x^4 - 6x^3 - 39x^2 - 18x$

Multiply the dividend by 3:

$$\begin{array}{r|l} & 4 \\ 18x^4 - 6x^3 - 39x^2 - 18x & 24x^4 - 2x^3 - 60x^2 - 32x \\ & \times 3 \\ & 72x^4 - 6x^3 - 180x^2 - 96x \\ & -(72x^4 - 24x^3 - 156x^2 - 72x) \\ & \hline & 18x^3 - 24x^2 - 24x \end{array}$$

$$= 6(3x^3 - 4x^2 - 4x)$$

Leave out 6 and divide by $3x^3 - 4x^2 - 4x$:

$$
\begin{array}{r|l}
3x^3 - 4x^2 - 4x & 18x^4 - 6x^3 - 39x^2 - 18x \\
 & \quad 6x + 6 \\
 & -(18x^4 - 24x^3 - 24x^2) \\
\hline
 & \quad 18x^3 - 15x^2 - 18x \\
 & -(18x^3 - 24x^2 - 24x) \\
\hline
 & \quad 9x^2 + 6x
\end{array}
$$

$= 3(3x^2 + 2x)$

Leave out 3 and divide by $3x^2 + 2x$:

$$
\begin{array}{r|l}
3x^2 + 2x & 3x^3 - 4x^2 - 4x \\
 & \quad x - 2 \\
 & -(3x^3 + 2x^2) \\
\hline
 & \quad -6x^2 - 4x \\
 & -(-6x^2 - 4x) \\
\hline
 & \quad 0
\end{array}
$$

$\therefore$ Required HCF $= 3x^2 + 2x = x(3x + 2)$

8) $2x^3 + 6x^2 + x + 3,\ 3x^3 + 9x^2 - 2x - 6,\ x^3 + 3x^2 + 2x + 6$

First find the HCF of the second and third polynomials.

$$
\begin{array}{r|l}
x^3 + 3x^2 + 2x + 6 & 3x^3 + 9x^2 - 2x - 6 \\
 & \quad 3 \\
 & -(3x^3 + 9x^2 + 6x + 18) \\
\hline
 & \quad -8x - 24
\end{array}
$$

$= -8(x + 3)$

Leave out -8 and divide by $x + 3$:

$$
\begin{array}{r|l}
x + 3 & x^2 + 2 \\
\hline
 & x^3 + 3x^2 + 2x + 6 \\
 & -(x^3 + 3x^2) \\
\hline
 & 2x + 6 \\
 & -(2x + 6) \\
\hline
 & 0
\end{array}
$$

So $x + 3$ is the HCF of these two. Now divide the first polynomial by $x + 3$:

$$
\begin{array}{r|l}
x + 3 & 2x^2 + 1 \\
\hline
 & 2x^3 + 6x^2 + x + 3 \\
 & -(2x^3 + 6x^2) \\
\hline
 & x + 3 \\
 & -(x + 3) \\
\hline
 & 0
\end{array}
$$

$\therefore$ Required HCF $= x + 3$

Find the LCM of the following expressions.

9) $9a^2b - b,\ 6a^2 + 2a$

$9a^2b - b = b(9a^2 - 1) = b(3a + 1)(3a - 1)$

$6a^2 + 2a = 2a(3a + 1)$

Factors with highest powers: $2a,\ b,\ 3a + 1,\ 3a - 1$

LCM $= 2a \cdot b(3a + 1)(3a - 1)$

$\therefore$ Required LCM $= 2ab(9a^2 - 1)$

10) $p^3q - pq^3,\ p^5q^2 - p^2q^5$

$p^3q - pq^3 = pq(p^2 - q^2) = pq(p + q)(p - q)$

$p^5q^2 - p^2q^5 = p^2q^2(p^3 - q^3) = p^2q^2(p - q)(p^2 + pq + q^2)$

Factors with highest powers: $p^2,\ q^2,\ p - q,\ p + q,\ p^2 + pq + q^2$

LCM $= p^2q^2(p - q)(p + q)(p^2 + pq + q^2)$

$\therefore$ Required LCM $= p^2q^2(p + q)(p^3 - q^3)$

11) $4x^2y - y,\ 2x^2 + x$

$4x^2y - y = y(4x^2 - 1) = y(2x + 1)(2x - 1)$

$2x^2 + x = x(2x + 1)$

Factors with highest powers: $x,\ y,\ 2x + 1,\ 2x - 1$

$LCM = xy(2x + 1)(2x - 1)$

$\therefore$ Required LCM $= xy(4x^2 - 1)$

12) $x^2 - x - 6,\ x^2 + x - 2,\ x^2 - 4x + 3$

$x^2 - x - 6 = x^2 - 3x + 2x - 6 = (x - 3)(x + 2)$

$x^2 + x - 2 = x^2 + 2x - x - 2 = (x + 2)(x - 1)$

$x^2 - 4x + 3 = x^2 - 3x - x + 3 = (x - 3)(x - 1)$

Factors with highest powers: $x - 3,\ x - 1,\ x + 2$

$\therefore$ Required LCM $= (x - 1)(x - 3)(x + 2)$

13) $m^6 - 1,\ m^4 - 1,\ m^3 - 1$

$m^6 - 1 = (m^2)^3 - (1)^3 = (m^2 - 1)(m^4 + m^2 + 1)$

$= (m + 1)(m - 1)(m^2 + m + 1)(m^2 - m + 1)$

$m^4 - 1 = (m^2 + 1)(m^2 - 1) = (m + 1)(m - 1)(m^2 + 1)$

$m^3 - 1 = (m - 1)(m^2 + m + 1)$

Factors with highest powers: $m - 1,\ m + 1,\ m^2 + 1,\ m^2 + m + 1,\ m^2 - m + 1$

$LCM = (m - 1)(m + 1)(m^2 + 1)(m^2 + m + 1)(m^2 - m + 1)$

$= (m^2 + 1)(m^3 - 1)(m^3 + 1)$

$\therefore$ Required LCM $= (m^2 + 1)(m^6 - 1)$

14) $x^3 + 2x^2 - x - 2,\ x^2 - x - 2,\ x^2 - 4$

$x^3 + 2x^2 - x - 2 = x^2(x + 2) - 1(x + 2) = (x + 2)(x^2 - 1)$

$= (x + 1)(x - 1)(x + 2)$

$x^2 - x - 2 = x^2 - 2x + x - 2 = (x - 2)(x + 1)$

$x^2 - 4 = (x + 2)(x - 2)$

Factors with highest powers: $x + 1,\ x - 1,\ x + 2,\ x - 2$

$LCM = (x + 1)(x - 1)(x + 2)(x - 2) = (x^2 - 1)(x^2 - 4)$

$\therefore$ Required LCM $= x^4 - 5x^2 + 4$

15) $x^2 + x - 20$, $x^2 - 10x + 24$, $x^2 - x - 30$

$x^2 + x - 20 = x^2 + 5x - 4x - 20 = (x + 5)(x - 4)$

$x^2 - 10x + 24 = x^2 - 4x - 6x + 24 = (x - 4)(x - 6)$

$x^2 - x - 30 = x^2 - 6x + 5x - 30 = (x - 6)(x + 5)$

Factors with highest powers: $x + 5$, $x - 4$, $x - 6$

$\therefore$ Required LCM $= (x + 5)(x - 4)(x - 6)$