Exercise 4.5

Unit 4 · Factorization and Algebraic Manipulation

1 Find the HCF and LCM of the following.

(i) $16 - 4x^2$, $x^2 + x - 6$

Let $P = 16 - 4x^2 = 4(4 - x^2) = 4[(2)^2 - (x)^2]$

$= 4(2 + x)(2 - x) = -4(x + 2)(x - 2)$

and $Q = x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= x(x + 3) - 2(x + 3) = (x + 3)(x - 2)$

$\therefore$ HCF of P and Q $= x - 2$

LCM of P and Q $= -4(x + 3)(x - 2)(x + 2)$

$\therefore$ LCM $= -4(x + 3)(x^2 - 4)$

(ii) $a^4 - a^3 - a + 1$, $a^4 + a^2 + 1$

Let $P = a^4 - a^3 - a + 1 = a^3(a - 1) - 1(a - 1)$

$= (a^3 - 1)(a - 1) = (a - 1)(a^2 + a + 1)(a - 1)$

$= (a - 1)^2(a^2 + a + 1)$

and $Q = a^4 + a^2 + 1 = (a^2)^2 + (1)^2 + 2a^2 - 2a^2 + a^2$

$= (a^2 + 1)^2 - a^2 = (a^2 + 1 - a)(a^2 + 1 + a)$

$\therefore$ HCF of P and Q $= a^2 + a + 1$

LCM $= (a - 1)^2(a^2 + a + 1)(a^2 - a + 1)$

$\therefore$ LCM $= (a - 1)(a^3 - 1)(a^2 - a + 1)$

(iii) $x^3 + 2x^2 - 3x$, $2x^3 + 5x^2 - 3x$

Let $P = x^3 + 2x^2 - 3x = x(x^2 + 2x - 3) = x(x^2 + 3x - x - 3)$

$= x[x(x + 3) - 1(x + 3)] = x(x + 3)(x - 1)$

and $Q = 2x^3 + 5x^2 - 3x = x(2x^2 + 5x - 3)$

$= x(2x^2 + 6x - x - 3)$

$= x[2x(x + 3) - 1(x + 3)] = x(x + 3)(2x - 1)$

$\therefore$ HCF of P and Q $= x(x + 3)$

$\therefore$ LCM of P and Q $= x(x + 3)(x - 1)(2x - 1)$

2 **If the HCF and LCM of two polynomials are $x - 7$ and $x^3 - 10x^2 + 11x + 70$ respectively, then find the product of the two polynomials.**

Product of two polynomials = HCF × LCM

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$= (x^3 - 10x^2 + 11x + 70)(x - 7)$

$= x^4 - 7x^3 - 10x^3 + 70x^2 + 11x^2 - 77x + 70x - 490$

$\therefore$ Product $= x^4 - 17x^3 + 81x^2 - 7x - 490$

3 **The product of two polynomials is $x^4 + 3x^3 - 12x^2 - 20x + 48$ and their HCF is $x - 2$. Find their LCM.**

$P \times Q = x^4 + 3x^3 - 12x^2 - 20x + 48$, $HCF = x - 2$

By the relation $P \times Q = HCF \times LCM$: $LCM = \dfrac{P \times Q}{HCF}$

$$LCM = \frac{x^4 + 3x^3 - 12x^2 - 20x + 48}{x - 2}$$

$$\begin{array}{r|l} & x^3 + 5x^2 - 2x - 24 \\ \hline x - 2 & x^4 + 3x^3 - 12x^2 - 20x + 48 \\ & -(x^4 - 2x^3) \\ \hline & \quad 5x^3 - 12x^2 - 20x + 48 \\ & \quad -(5x^3 - 10x^2) \\ \hline & \quad\quad -2x^2 - 20x + 48 \\ & \quad\quad -(-2x^2 + 4x) \\ \hline & \quad\quad\quad -24x + 48 \\ & \quad\quad\quad -(-24x + 48) \\ \hline & \quad\quad\quad\quad 0 \end{array}$$

$\therefore LCM = x^3 + 5x^2 - 2x - 24$

4 **The product of two polynomials is $y^4 + 6y^3 - 3y^2 - 56y - 48$ and their LCM is $y^3 + 2y^2 - 11y - 12$. Find their HCF.**

Product = HCF × LCM

$$HCF = \frac{\text{Product}}{LCM} = \frac{y^4 + 6y^3 - 3y^2 - 56y - 48}{y^3 + 2y^2 - 11y - 12}$$

$$\begin{array}{r|l} & y + 4 \\ \hline y^3 + 2y^2 - 11y - 12 & y^4 + 6y^3 - 3y^2 - 56y - 48 \\ & -(y^4 + 2y^3 - 11y^2 - 12y) \\ \hline & 4y^3 + 8y^2 - 44y - 48 \\ & -(4y^3 + 8y^2 - 44y - 48) \\ \hline & 0 \end{array}$$

$\therefore$ HCF = $y + 4$

5 **Find the second polynomial when the first polynomial is $x^4 + x^3 + x + 1$, HCF = $x + 1$ and LCM = $(x^3 + 1)(x^4 + x^3 - x - 1)$.**

Let $P = x^4 + x^3 + x + 1$ and the second polynomial be Q.

By the relation $P \times Q = HCF \times LCM$: $Q = \frac{HCF \times LCM}{P}$

$$Q = \frac{(x + 1)(x^3 + 1)(x^4 + x^3 - x - 1)}{x^4 + x^3 + x + 1}$$

$$= \frac{(x + 1)(x^3 + 1)(x^4 + x^3 - x - 1)}{x^3(x + 1) + 1(x + 1)}$$

$$= \frac{(x + 1)(x^3 + 1)(x^4 + x^3 - x - 1)}{(x + 1)(x^3 + 1)}$$

$\therefore$ Second polynomial $Q = x^4 + x^3 - x - 1$

6 **Find the LCM of the polynomials $4x^3 - 10x^2 + 4x + 2$ and $3x^4 - 2x^3 - 3x + 2$ if their HCF is $x - 1$.**

Let $P = 4x^3 - 10x^2 + 4x + 2$, $Q = 3x^4 - 2x^3 - 3x + 2$

HCF = $x - 1$

$$LCM = \frac{P \times Q}{HCF} = \frac{(4x^3 - 10x^2 + 4x + 2)(3x^4 - 2x^3 - 3x + 2)}{x - 1}$$

Divide Q by $x - 1$:

$$
\begin{array}{r|l}
 & 3x^3 + x^2 + x - 2 \\
\hline
x - 1 & 3x^4 - 2x^3 \quad\quad - 3x + 2 \\
 & -(3x^4 - 3x^3) \\
\hline
 & \quad x^3 \quad\quad - 3x + 2 \\
 & \quad -(x^3 - x^2) \\
\hline
 & \quad\quad x^2 - 3x + 2 \\
 & \quad\quad -(x^2 - x) \\
\hline
 & \quad\quad -2x + 2 \\
 & \quad\quad -(-2x + 2) \\
\hline
 & \quad\quad\quad 0
\end{array}
$$

So $Q \div (x - 1) = 3x^3 + x^2 + x - 2$

$\therefore$ LCM $= (4x^3 - 10x^2 + 4x + 2)(3x^3 + x^2 + x - 2)$