Exercise 4.6

Unit 4 · Factorization and Algebraic Manipulation

1 **Answers**

(i) $\frac{1}{x^3 + 7x - 8}$ (ii) $\frac{x^2}{x^2 + y^2}$

(iii) $\frac{n}{m^2 + n^2}$ (iv) 1

Simplify.

2) $\frac{14x^2 - 7x}{12x^3 + 24x^2} \times \frac{x^2 + 2x}{2x - 1}$

$= \frac{7x(2x - 1)}{12x^2(x + 2)} \times \frac{x(x + 2)}{2x - 1} = \frac{7x^2}{12x^2}$

$\therefore \frac{7}{12}$

3) $\frac{a^2b^2 + 3ab}{4a^2 - 1} \times \frac{2a + 1}{ab + 3}$

$= \frac{ab(ab + 3)}{(2a)^2 - (1)^2} \times \frac{2a + 1}{ab + 3}$

$= \frac{ab}{(2a + 1)(2a - 1)} \times \frac{2a + 1}{1}$

$\therefore \frac{ab}{2a - 1}$

4) $\dfrac{a-b}{a^2+ab} \times \dfrac{a^4-b^4}{a^2-2ab+b^2} \times \dfrac{a}{a^2+b^2}$

$= \dfrac{a-b}{a(a+b)} \times \dfrac{(a^2-b^2)(a^2+b^2)}{(a-b)^2} \times \dfrac{a}{a^2+b^2}$

$= \dfrac{a-b}{a(a+b)} \times \dfrac{(a+b)(a-b)(a^2+b^2)}{(a-b)^2} \times \dfrac{a}{a^2+b^2}$

All the factors cancel.

$\therefore 1$

5) $\dfrac{6x^2y^2}{x^2-y^2} \div \dfrac{3xy}{x+y}$

$= \dfrac{6x^2y^2}{(x+y)(x-y)} \div \dfrac{3xy}{x+y}$

$= \dfrac{6x^2y^2}{(x+y)(x-y)} \times \dfrac{x+y}{3xy}$

$\therefore \dfrac{2xy}{x-y}$

6) $\dfrac{8a^3-1}{4a^3+2a^2} \div \dfrac{6a^2-13a+5}{15a-25} \times \dfrac{2a^4+a^3}{15a^2}$

$= \dfrac{(2a)^3-(1)^3}{2a^2(2a+1)} \div \dfrac{6a^2-10a-3a+5}{5(3a-5)} \times \dfrac{a^3(2a+1)}{15a^2}$

$= \dfrac{(2a-1)(4a^2+2a+1)}{2a^2(2a+1)} \times \dfrac{5(3a-5)}{2a(3a-5)-1(3a-5)}$

$\times \dfrac{a^3(2a+1)}{15a^2}$

$= \dfrac{(2a-1)(4a^2+2a+1)}{2a^2} \times \dfrac{5(3a-5)}{(3a-5)(2a-1)} \times \dfrac{a^3}{15a^2}$

$= \dfrac{(4a^2+2a+1)(a)}{6a^2}$

$\therefore \dfrac{4a^2+2a+1}{6a}$

7) $$\frac{x^2-8x-9}{x^2-17x+72} \times \frac{x^2-25}{x^2-1} \div \frac{x^2+4x-5}{x^2-9x+8}$$

$$= \frac{(x-9)(x+1)}{x^2-9x-8x+72} \times \frac{(x+5)(x-5)}{(x+1)(x-1)} \times \frac{x^2-8x-x+8}{x^2+5x-x-5}$$

$$= \frac{(x-9)(x+1)}{(x-9)(x-8)} \times \frac{(x+5)(x-5)}{(x+1)(x-1)} \times \frac{(x-8)(x-1)}{(x+5)(x-1)}$$

$$\therefore \frac{x-5}{x-1}$$

8) $$\frac{1}{2x-3y} - \frac{x+y}{4x^2-9y^2}$$

$$= \frac{1}{2x-3y} - \frac{x+y}{(2x+3y)(2x-3y)}$$

$$= \frac{(2x+3y)-(x+y)}{(2x+3y)(2x-3y)} = \frac{2x+3y-x-y}{4x^2-9y^2}$$

$$\therefore \frac{x+2y}{4x^2-9y^2}$$

9) $$\frac{1}{x(x-y)} + \frac{1}{y(x+y)}$$

$$= \frac{y(x+y)+x(x-y)}{xy(x-y)(x+y)} = \frac{xy+y^2+x^2-xy}{xy(x-y)(x+y)}$$

$$\therefore \frac{x^2+y^2}{xy(x^2-y^2)}$$

10) $\dfrac{5x+5}{3(2x-1)} + \dfrac{6-2x}{2(1-2x)}$

$= \dfrac{5x+5}{3(2x-1)} - \dfrac{6-2x}{2(2x-1)}$ [since $1-2x = -(2x-1)$]

$= \dfrac{2(5x+5) - 3(6-2x)}{6(2x-1)} = \dfrac{10x+10-18+6x}{6(2x-1)}$

$= \dfrac{16x-8}{6(2x-1)} = \dfrac{8(2x-1)}{6(2x-1)}$

$\therefore \dfrac{4}{3}$

11) $\dfrac{2a}{2a-3} - \dfrac{5}{6a+9} - \dfrac{4(3a+2)}{3(4a^2-9)}$

$= \dfrac{2a}{2a-3} - \dfrac{5}{3(2a+3)} - \dfrac{4(3a+2)}{3(2a+3)(2a-3)}$

$= \dfrac{2a(3)(2a+3) - 5(2a-3) - 4(3a+2)}{3(2a+3)(2a-3)}$

$= \dfrac{12a^2+18a-10a+15-12a-8}{3(4a^2-9)}$

$\therefore \dfrac{12a^2-4a+7}{3(4a^2-9)}$

12) $\dfrac{5}{5 + x - 18x^2} - \dfrac{2}{2 + 5x + 2x^2}$

$= \dfrac{-5}{18x^2 - x - 5} - \dfrac{2}{2x^2 + 5x + 2}$

$= \dfrac{-5}{18x^2 - 10x + 9x - 5} - \dfrac{2}{2x^2 + 4x + x + 2}$

$= \dfrac{-5}{2x(9x - 5) + 1(9x - 5)} - \dfrac{2}{2x(x + 2) + 1(x + 2)}$

$= \dfrac{-5}{(2x + 1)(9x - 5)} - \dfrac{2}{(x + 2)(2x + 1)}$

$= \dfrac{-5(x + 2) - 2(9x - 5)}{(2x + 1)(9x - 5)(x + 2)} = \dfrac{-5x - 10 - 18x + 10}{(2x + 1)(9x - 5)(x + 2)}$

$= \dfrac{-23x}{(2x + 1)(9x - 5)(x + 2)}$

$\therefore \dfrac{23x}{(1 + 2x)(2 + x)(5 - 9x)}$

13) $\dfrac{1 - p^2}{1 + q} \times \dfrac{1 - q^2}{p + p^2} \times \left(1 + \dfrac{p}{1 - p}\right)$

$= \dfrac{(1 + p)(1 - p)}{1 + q} \times \dfrac{(1 + q)(1 - q)}{p(1 + p)} \times \dfrac{1 - p + p}{1 - p}$

$= \dfrac{(1 + p)(1 - p)}{1 + q} \times \dfrac{(1 + q)(1 - q)}{p(1 + p)} \times \dfrac{1}{1 - p}$

$\therefore \dfrac{1 - q}{p}$