ExamPath

CLASS 9 MATHEMATICS · FBISE · UNIT 4 · SECTIONS B AND C PRACTICE

Unit 4: Factorization and Algebraic Manipulation: short and long questions

Write your working in the boxes. Model answers are on the last pages. These are practice questions written by ExamPath, not official FBISE questions or marking schemes.

Section B style: short questions (3 marks each)

B1 · 3 marks · Factorising quadratics

Factorise $2x^2 + 7x + 3$.

B2 · 3 marks · Factor theorem

Use the factor theorem to show that $(x - 2)$ is a factor of $x^3 - 3x^2 - 4x + 12$, and then factorise it completely.

B3 · 3 marks · Remainder theorem

Find the remainder when $2x^3 + 3x^2 - 5x + 4$ is divided by $(x + 2)$.

B4 · 3 marks · Square roots of expressions

Find the square root of $4x^2 - 12x + 9$.

B5 · 3 marks · Algebraic fractions

Simplify $(x^2 - 9)/(x^2 + 5x + 6)$.

B6 · 3 marks · HCF and LCM

Find the HCF and LCM of $12x^2y^3$ and $18x^3y$.

Section C style: long questions (6 marks each)

C1 · 6 marks · HCF and LCM by factorisation

Let $p(x) = x^2 - 5x + 6$ and $q(x) = x^2 - x - 6$.

(a) Factorise $p(x)$ and $q(x)$. (2 marks)

(b) Find their HCF and LCM. (2 marks)

(c) Verify that $p(x) \times q(x) = \text{HCF} \times \text{LCM}$. (2 marks)

C2 · 6 marks · Factorising cubics in real life

A box has volume $V = x^3 + 6x^2 + 11x + 6$ cm^3.

(a) Show that $(x + 1)$ is a factor of V. (2 marks)

(b) Factorise V completely to find expressions for the three dimensions. (2 marks)

(c) Find the dimensions and the volume when $x = 4$. (2 marks)

C3 · 6 marks · Algebraic identities

If $a + b = 7$ and $ab = 10$, find

(a) $a^2 + b^2$ (2 marks)

(b) $a^3 + b^3$ (2 marks)

(c) $a - b$, given that $a > b$ (2 marks)

Model answers

B1.

1. Multiply the first and last coefficients: $2 \times 3 = 6$. Find two numbers with product 6 and sum 7: 6 and 1.
2. Split the middle term: $2x^2 + 6x + x + 3$
3. Group: $2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$

Answer: $(2x + 1)(x + 3)$

Watch out: Check by expanding: $(2x + 1)(x + 3) = 2x^2 + 6x + x + 3 = 2x^2 + 7x + 3$.

B2.

1. Put $x = 2$: $8 - 12 - 8 + 12 = 0$, so $(x - 2)$ is a factor.
2. Group the terms: $x^2(x - 3) - 4(x - 3) = (x - 3)(x^2 - 4)$
3. $x^2 - 4 = (x - 2)(x + 2)$, so the full factorisation is $(x - 2)(x + 2)(x - 3)$.

Answer: $(x - 2)(x + 2)(x - 3)$

Watch out: For $(x - 2)$ substitute $x = +2$, not -2.

B3.

1. By the remainder theorem, substitute $x = -2$.
2. $2(-8) + 3(4) - 5(-2) + 4 = -16 + 12 + 10 + 4 = 10$

Answer: Remainder = 10

Watch out: Take care with signs: $(-2)^3 = -8$ and $-5 \times (-2) = +10$.

B4.

1. $4x^2 = (2x)^2$ and $9 = 3^2$, and $2 \times 2x \times 3 = 12x$.
2. So $4x^2 - 12x + 9 = (2x - 3)^2$.
3. The square root is $\pm(2x - 3)$.

Answer: $\pm(2x - 3)$

Watch out: Check the middle term: a perfect square $a^2 - 2ab + b^2$ needs $2ab = 12x$.

B5.

1. $x^2 - 9 = (x - 3)(x + 3)$
2. $x^2 + 5x + 6 = (x + 2)(x + 3)$
3. Cancel the common factor $(x + 3)$: the result is $(x - 3)/(x + 2)$.

Answer: $(x - 3)/(x + 2)$

Watch out: Only cancel whole factors; x^2 cannot be cancelled from the top and bottom.

B6.

1. Numbers: HCF(12, 18) = 6 and LCM(12, 18) = 36.
2. HCF uses the lowest powers: x^2 and y. So HCF = $6x^2y$.
3. LCM uses the highest powers: x^3 and y^3. So LCM = $36x^3y^3$.

Answer: HCF = $6x^2y$, LCM = $36x^3y^3$

Watch out: HCF takes the lowest power of each variable, LCM takes the highest.

C1.

1. (a) $p(x) = (x - 2)(x - 3)$ and $q(x) = (x - 3)(x + 2)$.
2. (b) HCF = the common factor = $x - 3$. LCM = $(x - 2)(x - 3)(x + 2)$.
3. (c) $p(x) \times q(x) = (x - 2)(x - 3)(x - 3)(x + 2)$.
4. HCF × LCM = $(x - 3) \times (x - 2)(x - 3)(x + 2) = (x - 2)(x - 3)(x - 3)(x + 2)$. Both are the same product, so the result is verified.

Answer: HCF = $x - 3$; LCM = $(x - 2)(x - 3)(x + 2)$

Watch out: The LCM uses each factor once, at its highest power; do not repeat $(x - 3)$.

C2.

1. (a) Put $x = -1$: $-1 + 6 - 11 + 6 = 0$, so $(x + 1)$ is a factor.
2. (b) Divide: $x^3 + 6x^2 + 11x + 6 = (x + 1)(x^2 + 5x + 6) = (x + 1)(x + 2)(x + 3)$.
3. (c) When $x = 4$ the dimensions are 5 cm, 6 cm and 7 cm.
4. Volume $= 5 \times 6 \times 7 = 210$ cm^3. Check: $64 + 96 + 44 + 6 = 210$.

Answer: (b) $(x + 1)(x + 2)(x + 3)$ (c) 5 cm, 6 cm, 7 cm; 210 cm^3

Watch out: After finding one factor, divide (or compare coefficients) to get the quadratic, then factorise that.

C3.

1. (a) $a^2 + b^2 = (a + b)^2 - 2ab = 49 - 20 = 29$.
2. (b) $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = 343 - 3 \times 10 \times 7 = 343 - 210 = 133$.
3. (c) $(a - b)^2 = a^2 + b^2 - 2ab = 29 - 20 = 9$, so $a - b = 3$ (positive because $a > b$).
4. Check: $a = 5$ and $b = 2$ give $25 + 4 = 29$ and $125 + 8 = 133$.

Answer: (a) 29 (b) 133 (c) 3

Watch out: Use the identities; you do not need to find a and b first, though it is a good way to check.

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