Exercise 6.2

Unit 6 · Trigonometry and Bearing

Arc length: $l = r\theta$ Area of sector: $A = \frac{1}{2}r^2\theta$

θ must be in radians; take $\pi = 3.1416$.

1 Find the length of the arc and the area of the sector.

(i) $r = 5$ cm, $\theta = \frac{\pi}{3}$ rad

$$l = r\theta = 5\left(\frac{\pi}{3}\right) = \frac{5}{3}(3.1416)$$

$\therefore l = 5.24$ cm

$$A = \frac{1}{2}(5)^2\left(\frac{\pi}{3}\right) = \frac{25}{6}(3.1416)$$

$\therefore A = 13.09\ cm^2$

(ii) $r = 12$ m, $\theta = 120° = \frac{2\pi}{3}$ rad

$$l = 12\left(\frac{2\pi}{3}\right) = 8\pi$$

$\therefore l = 25.13$ m

$$A = \frac{1}{2}(12)^2\left(\frac{2\pi}{3}\right) = 48\pi = 48(3.1416)$$

$\therefore A = 150.80\ m^2$

(iii) $r = 6$ dm, $\theta = 60°45'30''$

$$\theta = 60° + \left(\frac{45}{60}\right)° + \left(\frac{30}{3600}\right)° = 60.7583°$$

$$\theta = 60.7583 \times \frac{\pi}{180} = 0.3375\pi \text{ rad}$$

$$l = 6(0.3375)(3.1416)$$

$\therefore l = 6.36$ dm

$$A = \frac{1}{2}(6)^2(0.3375)(3.1416)$$

$\therefore A = 19.08$ dm^2

2 **A circle has radius 4 cm and a central angle of 75°. Find the arc length and the area of the sector.**

$$\theta = 75 \times \frac{\pi}{180} = \frac{5\pi}{12} \text{ rad}$$

$$l = r\theta = 4\left(\frac{5\pi}{12}\right) = \frac{5}{3}(3.1416)$$

$\therefore l = 5.24$ cm

$$A = \frac{1}{2}(4)^2\left(\frac{5\pi}{12}\right) = \frac{10}{3}(3.1416)$$

$\therefore A = 10.47$ cm^2

3 **Find the radian measure of the central angle.**

(i) $l = 8$ cm, $r = 4$ cm

$$\theta = \frac{l}{r} = \frac{8}{4}$$

$\therefore \theta = 2$ rad

(ii) $l = 8.5$ m, $r = 2.25$ m

$$\theta = \frac{l}{r} = \frac{8.5}{2.25}$$

$\therefore \theta = 3.78$ rad

(iii) $A = 45\ cm^2,\ r = 10.7\ cm$

$$A = \frac{1}{2}r^2\theta \Rightarrow 45 \times 2 = (10.7)^2\theta$$

$$\theta = \frac{90}{114.49}$$

$\therefore \theta = 0.786\ rad$

(iv) $A = 100\ cm^2,\ l = 10\ cm$

$$l = r\theta \Rightarrow \theta = \frac{10}{r} \quad ...(i)$$

$$A = \frac{1}{2}r^2\left(\frac{10}{r}\right) = 5r \Rightarrow 100 = 5r \Rightarrow r = 20\ cm$$

From (i): $\theta = \frac{10}{20}$

$\therefore \theta = 0.5\ rad$

4 Find the radius.

(i) $l = 4\ cm,\ \theta = \pi\ rad$

$$r = \frac{l}{\theta} = \frac{4}{\pi} = \frac{4}{3.1416}$$

$\therefore r = 1.27\ cm$

(ii) $l = 6\ m,\ \theta = 15° = 15 \times \frac{\pi}{180} = \frac{\pi}{12}\ rad$

$$r = \frac{l}{\theta} = \frac{6}{\frac{\pi}{12}} = \frac{72}{\pi}$$

$\therefore r = 22.92\ m$

(iii) $A = 200\ cm^2,\ \theta = \frac{\pi}{4}\ rad$

$$r^2 = \frac{2A}{\theta} = \frac{2(200)}{\frac{\pi}{4}} = \frac{1600}{\pi} = 509.29$$

$\therefore r = 22.57\ cm$

(iv) $A = 100\ dm^2,\ l = 10\ dm$

$$\theta = \frac{l}{r},\ \text{so } A = \frac{1}{2}r^2\left(\frac{l}{r}\right) = \frac{1}{2}lr \Rightarrow r = \frac{2A}{l}$$

$$r = \frac{2(100)}{10}$$

$\therefore r = 20\ dm$

5 **A pendulum of length 30 inches swings through 30°. Find the length of the arc it moves along.**

$$r = 30\ in,\ \theta = 30 \times \frac{\pi}{180} = \frac{\pi}{6}\ rad$$

$$l = r\theta = 30\left(\frac{\pi}{6}\right) = 5\pi$$

$\therefore l = 15.71$ inches

6 **A car travels at 42 km/h for 21 minutes on a circular track of radius 10 km. Find the angle it turns through, in degrees.**

$$\text{Time} = 21\ min = \frac{21}{60}\ h = 0.35\ h$$

Distance (arc) l = speed × time = $42 \times 0.35 = 14.7$ km

$$\theta = \frac{l}{r} = \frac{14.7}{10} = 1.47\ rad = 1.47 \times \frac{180}{\pi}°$$

$\therefore \theta = 84.2° \approx 84°$

7 **Find the perimeter and the area of a semicircle of diameter 12 cm.**

$d = 12\ cm,\ r = 6\ cm,\ \theta = \pi\ rad$

Perimeter = arc + diameter $= r\theta + d = 6\pi + 12$

$\therefore$ Perimeter $= 30.85$ cm

$$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2}(6)^2(\pi) = 18\pi$$

$\therefore$ Area $= 56.55\ cm^2$

8 **Find the angle between the hour hand and the minute hand of a clock, in radians.**

A clock face has 12 equal parts, so each part $= \frac{360°}{12} = 30°$.

(i) 9 o'clock: 3 parts apart $= 3 \times 30° = 90° = 90 \times \frac{\pi}{180}$

$\therefore \frac{\pi}{2}$ rad

(ii) 2:30: the hour hand is halfway between 2 and 3, so the hands are 3.5 parts apart.

$3.5 \times 30° = 105° = 105 \times \frac{\pi}{180}$

$\therefore \frac{7\pi}{12}$ rad

(iii) 6:45: the hour hand is three-quarters of the way from 6 to 7, so the hands are 2.25 parts apart.

$2.25 \times 30° = 67.5° = \frac{135}{2} \times \frac{\pi}{180}$

$\therefore \frac{3\pi}{8}$ rad