Exercise 6.3

Unit 6 · Trigonometry and Bearing

2 **Find the value of each expression.**

(i) $\sin 60° - \cos 30°$

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$

$\therefore 0$

(ii) $\sec 45° \operatorname{cosec} 45° - \sin 30° \cos 60°$

$$= (\sqrt{2})(\sqrt{2}) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 2 - \frac{1}{4} = \frac{8-1}{4}$$

$\therefore \frac{7}{4}$

(iii) $$\frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{6}}$$

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + (\sqrt{3})\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{2}{\sqrt{3}}}{2}$$

$\therefore \frac{1}{\sqrt{3}}$

(iv) $\cos^2\frac{\pi}{3} - \sin^2\frac{\pi}{3}$

$$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4}$$

$\therefore -\frac{1}{2}$

(v) $\sin\frac{\pi}{2}\cos\frac{\pi}{3} - \cos\frac{\pi}{2}\sin\frac{\pi}{3}$

$= (1)\left(\frac{1}{2}\right) - (0)\left(\frac{\sqrt{3}}{2}\right)$

$\therefore \frac{1}{2}$

3 **Verify the identities for $\theta = \frac{\pi}{6}$.**

(i) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$L.H.S = \tan 2\left(\frac{\pi}{6}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

$R.H.S = \frac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\frac{1}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3}$

$\therefore$ L.H.S = R.H.S, verified

(ii) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$L.H.S = \sin 3\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{2} = 1$

$R.H.S = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3 = \frac{3}{2} - \frac{4}{8} = 1$

$\therefore$ L.H.S = R.H.S, verified

(iii) $\cos 2\theta = 1 - 2\sin^2\theta$

$L.H.S = \cos\frac{\pi}{3} = \frac{1}{2}$

$R.H.S = 1 - 2\left(\frac{1}{2}\right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore$ L.H.S = R.H.S, verified

(iv) $\sec^2\theta - \tan^2\theta = 1$

$$L.H.S = \left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3} - \frac{1}{3} = 1 = R.H.S$$

$\therefore$ Verified

(v) $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$$L.H.S = (\operatorname{cosec}\frac{\pi}{6})^2 = (2)^2 = 4$$

$$R.H.S = 1 + (\cot\frac{\pi}{6})^2 = 1 + (\sqrt{3})^2 = 1 + 3 = 4$$

$\therefore$ L.H.S = R.H.S, verified

4 **Find θ ($0° \leq \theta \leq 360°$, the angle used in the notes).**

(i) $\sin\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$$\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$\therefore \theta = 45°$

(ii) $\operatorname{cosec}\theta = -1$

$\sin\theta = -1 \Rightarrow \theta = 270°$

$\therefore \theta = 270°$

(iii) $\tan\theta = 1$

$$\theta = \tan^{-1}(1)$$

$\therefore \theta = 45°$

(iv) $\sec\theta = \frac{\sqrt{12}}{3}$

$$\sec\theta = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

$\therefore \theta = 30°$

(v) $\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\therefore \theta = 90°$

Each answer is the acute or quadrantal angle; e.g. $\sin\theta = 1/\sqrt{2}$ is also true for 135°.

5 **$\cos\theta = \frac{1}{2}$, terminal ray in the 1st quadrant. Find the other ratios.**

$\cos\theta = \frac{\text{Base}}{\text{Hyp}} = \frac{1}{2} \Rightarrow B = 1, H = 2$

$P^2 = H^2 - B^2 = 4 - 1 = 3 \Rightarrow P = \sqrt{3}$

In the 1st quadrant all ratios are positive:

$\therefore \sin\theta = \frac{\sqrt{3}}{2}, \quad \tan\theta = \sqrt{3}, \quad \sec\theta = 2$

$\therefore \operatorname{cosec}\theta = \frac{2}{\sqrt{3}}, \quad \cot\theta = \frac{1}{\sqrt{3}}$

6 **cosec θ = 2 and the terminal ray of θ is in the 2nd quadrant. Find cos θ $\left(\frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta + \cos^2\theta}\right)$.**

$\sin\theta = \frac{1}{2} = \frac{P}{H} \Rightarrow P = 1, H = 2$

$B^2 = 4 - 1 = 3 \Rightarrow B = \sqrt{3}$

In the 2nd quadrant cos θ is negative: $\cos\theta = -\frac{\sqrt{3}}{2}$

$\sin^2\theta - \cos^2\theta = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$

$\sin^2\theta + \cos^2\theta = \frac{1}{4} + \frac{3}{4} = 1$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{-\frac{1}{2}}{1}\right) = \frac{\sqrt{3}}{4}$

$\therefore \frac{\sqrt{3}}{4}$

Correction: the notes take cos θ = +√3/2, which gives −√3/4. In the 2nd quadrant cos θ is negative, so the answer is +√3/4.

7 **tan θ = −1, terminal ray not in the 2nd quadrant. Find the other ratios.**

tan θ is negative only in quadrants 2 and 4.
So θ is in the 4th quadrant (sin −, cos +).

$\tan\theta = \frac{P}{B} = \frac{-1}{1} \Rightarrow P = -1, B = 1$

$H^2 = 1 + 1 = 2 \Rightarrow H = \sqrt{2}$

$\therefore \sin\theta = -\frac{1}{\sqrt{2}}, \quad \cos\theta = \frac{1}{\sqrt{2}}, \quad \cot\theta = -1$

$\therefore \operatorname{cosec}\theta = -\sqrt{2}, \quad \sec\theta = \sqrt{2}$