ExamPath

CLASS 9 MATHEMATICS · FBISE · UNIT 6 · SECTIONS B AND C PRACTICE

Unit 6: Trigonometry and Bearing: short and long questions

Write your working in the boxes. Model answers are on the last pages. These are practice questions written by ExamPath, not official FBISE questions or marking schemes.

Section B style: short questions (3 marks each)

B1 · 3 marks · Degrees and radians

Convert (a) 225° to radians (b) 7π/12 radians to degrees (c) 2.5 radians to degrees, correct to 2 decimal places.

B2 · 3 marks · Arc length and sector area

A sector of a circle has radius 12 cm and central angle 150°. Find (a) its arc length (b) its area. Give answers in terms of π and to 1 decimal place.

B3 · 3 marks · Trigonometric identities

Prove that $(1 - \cos^2\theta)(1 + \cot^2\theta) = 1$.

B4 · 3 marks · Trigonometric ratios

If $\tan\theta = 5/12$ and θ is acute, find $\sin\theta$, $\cos\theta$ and $\sec\theta$.

B5 · 3 marks · Bearings

A ship sails 8 km due north and then 6 km due east. Find its distance from the starting point and its three-figure bearing from the start.

B6 · 3 marks · Special angles

Without a calculator, evaluate $2\sin 30^\circ \cos 60^\circ + \tan^2 45^\circ - \sin^2 60^\circ$.

Section C style: long questions (6 marks each)

C1 · 6 marks · Trigonometric identities

(a) Prove that $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$. (3 marks)

(b) Prove that $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$. (3 marks)

C2 · 6 marks · Angles of elevation

From a point P on level ground, 30 m from the foot of a tower, the angle of elevation of the top of the tower is 60°.

(a) Find the height of the tower. (2 marks)

(b) Find the distance from P to the top of the tower. (2 marks)

(c) Asad walks straight back from P until the angle of elevation is 30°. How far does he walk? (2 marks)

C3 · 6 marks · Bearings

Town B is 50 km from town A on a bearing of 060°. Town C is 50 km from A on a bearing of 150°.

(a) Find angle BAC. (2 marks)

(b) Find the distance BC. (2 marks)

(c) Find the bearing of A from B. (2 marks)

Model answers

B1.

1. (a) $225 \times \pi/180 = 5\pi/4$ radians.
2. (b) $7\pi/12 \times 180/\pi = 7 \times 15 = 105°$.
3. (c) $2.5 \times 180/\pi = 2.5 \times 57.2958\ldots \approx 143.24°$.

Answer: (a) $5\pi/4$ (b) $105°$ (c) $143.24°$

Watch out: To change degrees to radians multiply by $\pi/180$; to change radians to degrees multiply by $180/\pi$.

B2.

1. Change the angle to radians: $\theta = 150 \times \pi/180 = 5\pi/6$.
2. (a) $l = r\theta = 12 \times 5\pi/6 = 10\pi \approx 31.4$ cm.
3. (b) $A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 144 \times 5\pi/6 = 60\pi \approx 188.5$ cm^2.

Answer: (a) $10\pi \approx 31.4$ cm (b) $60\pi \approx 188.5$ cm^2

Watch out: $l = r\theta$ and $A = \frac{1}{2}r^2\theta$ only work when θ is in radians.

B3.

1. $1 - \cos^2\theta = \sin^2\theta$
2. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
3. LHS $= \sin^2\theta \times \operatorname{cosec}^2\theta = \sin^2\theta \times 1/\sin^2\theta = 1 =$ RHS

Answer: LHS = 1 = RHS

Watch out: Start from the more complicated side and use the three Pythagorean identities.

B4.

1. Draw a right triangle with opposite side 5 and adjacent side 12.
2. Hypotenuse $= \sqrt{(5^2 + 12^2)} = \sqrt{169} = 13$.
3. $\sin\theta = 5/13$, $\cos\theta = 12/13$, $\sec\theta = 1/\cos\theta = 13/12$.

Answer: $\sin\theta = 5/13$, $\cos\theta = 12/13$, $\sec\theta = 13/12$

Watch out: For an acute angle all the ratios are positive.

B5.

1. Distance $= \sqrt{(8^2 + 6^2)} = \sqrt{100} = 10$ km.
2. The angle east of north is θ with $\tan\theta = 6/8 = 0.75$, so $\theta \approx 36.9°$.
3. A bearing is measured clockwise from north, so the bearing is about 036.9° (037° to the nearest degree).

Answer: 10 km on a bearing of about 037°

Watch out: Measure bearings from north, clockwise, and write them with three figures.

B6.

1. $\sin 30° = 1/2$, $\cos 60° = 1/2$, $\tan 45° = 1$, $\sin 60° = \sqrt{3}/2$.
2. $2 \times 1/2 \times 1/2 = 1/2$; $\tan^2 45° = 1$; $\sin^2 60° = 3/4$.
3. $1/2 + 1 - 3/4 = 3/4$

Answer: 3/4

Watch out: $\tan^2 45°$ means $(\tan 45°)^2 = 1^2 = 1$. Square the value of the ratio, not the angle.

C1.

1. (a) Expand: $(\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta) + (\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta)$
2. The middle terms cancel: $2\sin^2\theta + 2\cos^2\theta = 2(\sin^2\theta + \cos^2\theta) = 2 \times 1 = 2$.
3. (b) LHS $= \sin\theta/\cos\theta + \cos\theta/\sin\theta = (\sin^2\theta + \cos^2\theta)/(\sin\theta\cos\theta)$
4. $= 1/(\sin\theta\cos\theta) = (1/\cos\theta)(1/\sin\theta) = \sec\theta\,\mathrm{cosec}\,\theta$ = RHS.

Answer: Both identities proved

Watch out: In (b), write everything in terms of sin θ and cos θ, then use a common denominator.

C2.

1. (a) $\tan 60° = h/30$, so $h = 30\tan 60° = 30\sqrt{3} \approx 51.96$ m.
2. (b) $\cos 60° = 30/d$, so $d = 30 \div 0.5 = 60$ m.
3. (c) At the new point, $\tan 30° = h/x$, so $x = 30\sqrt{3} \div (1/\sqrt{3}) = 30 \times 3 = 90$ m from the foot.
4. He walks $90 - 30 = 60$ m.

Answer: (a) $30\sqrt{3} \approx 51.96$ m (b) 60 m (c) 60 m

Watch out: Draw the diagram first and mark the right angle at the foot of the tower.

C3.

1. (a) Angle BAC $= 150° - 60° = 90°$.
2. (b) Triangle BAC is right-angled at A, so $BC = \sqrt{(50^2 + 50^2)} = 50\sqrt{2} \approx 70.71$ km.
3. (c) The back bearing is $060° + 180° = 240°$.

Answer: (a) 90° (b) $50\sqrt{2} \approx 70.71$ km (c) 240°

Watch out: To find the bearing of A from B, add 180° to the bearing of B from A (or subtract 180° if it is over 180°).

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