Exercise 7.1

Unit 7 · Coordinate Geometry

Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

1 Find the distance between each pair of points.

(i) $(-2, 1)$ and $(1, 5)$

$$d = \sqrt{(1 - (-2))^2 + (5 - 1)^2}$$
$$= \sqrt{9 + 16} = \sqrt{25} = 5$$

$\therefore d = 5$

(ii) $(1, 1)$ and $(7, 9)$

$$d = \sqrt{(7 - 1)^2 + (9 - 1)^2}$$
$$= \sqrt{36 + 64} = \sqrt{100} = 10$$

$\therefore d = 10$

(iii) $(0, -5)$ and $(-2, 1)$

$$d = \sqrt{(-2 - 0)^2 + (1 - (-5))^2}$$
$$= \sqrt{4 + 36} = \sqrt{40} = 2\sqrt{10}$$

$\therefore d = 2\sqrt{10}$

(iv) $(0, -5)$ and $(10, 5)$

$$d = \sqrt{(10 - 0)^2 + (5 - (-5))^2}$$
$$= \sqrt{100 + 100} = \sqrt{200} = 10\sqrt{2}$$

$\therefore d = 10\sqrt{2}$

(v) $(4, -1)$ and $(2, 0)$

$$d = \sqrt{(2 - 4)^2 + (0 - (-1))^2}$$
$$= \sqrt{4 + 1} = \sqrt{5}$$

$\therefore d = \sqrt{5}$

(vi) $(-3, 1)$ and $(-2, -3)$

$$d = \sqrt{(-2 - (-3))^2 + (-3 - 1)^2}$$

$$= \sqrt{1 + 16} = \sqrt{17}$$

$$\therefore d = \sqrt{17}$$

(vii) $\left(\frac{1}{2}, 2\right)$ and $(-2, 1)$

$$d = \sqrt{(-2 - \frac{1}{2})^2 + (1 - 2)^2}$$

$$= \sqrt{\frac{25}{4} + 1} = \sqrt{\frac{29}{4}} = \frac{\sqrt{29}}{2}$$

$$\therefore d = \frac{\sqrt{29}}{2}$$

(viii) $\left(\frac{7}{2}, 11\right)$ and $\left(13, \frac{17}{2}\right)$

$$d = \sqrt{(13 - \frac{7}{2})^2 + (\frac{17}{2} - 11)^2}$$

$$= \sqrt{\frac{361}{4} + \frac{25}{4}} = \sqrt{\frac{193}{2}} = \frac{\sqrt{386}}{2}$$

$$\therefore d = \frac{\sqrt{386}}{2}$$

(ix) $\left(-\frac{9}{4}, -\frac{7}{3}\right)$ and $\left(-\frac{3}{2}, -\frac{5}{4}\right)$

$$d = \sqrt{(-\frac{3}{2} - (-\frac{9}{4}))^2 + (-\frac{5}{4} - (-\frac{7}{3}))^2}$$

$$= \sqrt{\frac{9}{16} + \frac{169}{144}} = \sqrt{\frac{125}{72}} = \frac{5\sqrt{10}}{12}$$

$$\therefore d = \frac{5\sqrt{10}}{12}$$

(x) $(2\frac{1}{2}, 5\frac{1}{4})$ and $(5, -1)$ i.e. $(\frac{5}{2}, \frac{21}{4})$ and $(5, -1)$

$$d = \sqrt{(5 - \frac{5}{2})^2 + (-1 - \frac{21}{4})^2}$$

$$= \sqrt{\frac{25}{4} + \frac{625}{16}} = \sqrt{\frac{725}{16}} = \frac{5\sqrt{29}}{4}$$

$\therefore d = \frac{5\sqrt{29}}{4}$

In (viii), $\sqrt{\frac{193}{2}} = \frac{\sqrt{386}}{2}$ (multiply top and bottom inside the root by 2).

2 Check whether the following points are collinear or not by using the distance formula.

(i) $A(0, 1)$, $B(2, 3)$, $C(3, 4)$

$AB = \sqrt{(2 - 0)^2 + (3 - 1)^2}$

$= \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

$BC = \sqrt{(3 - 2)^2 + (4 - 3)^2}$

$= \sqrt{1 + 1} = \sqrt{2}$

$CA = \sqrt{(0 - 3)^2 + (1 - 4)^2}$

$= \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

Here $AB + BC = CA$: $2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$

$\therefore$ The given points are collinear.

(ii) $A(-5, -4)$, $B(1, 0)$, $C(6, 7)$

$AB = \sqrt{(1 - (-5))^2 + (0 - (-4))^2}$

$= \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}$

$BC = \sqrt{(6 - 1)^2 + (7 - 0)^2}$

$= \sqrt{25 + 49} = \sqrt{74}$

$CA = \sqrt{(-5 - 6)^2 + (-4 - 7)^2}$

$= \sqrt{121 + 121} = \sqrt{242} = 11\sqrt{2}$

Here $AB + BC \neq CA$: $2\sqrt{13} + \sqrt{74} \neq 11\sqrt{2}$

$\therefore$ The given points are not collinear.

(iii) $A(0, -5),\ B(3, 7),\ C(5, 15)$

$AB = \sqrt{(3-0)^2 + (7-(-5))^2}$

$= \sqrt{9 + 144} = \sqrt{153} = 3\sqrt{17}$

$BC = \sqrt{(5-3)^2 + (15-7)^2}$

$= \sqrt{4 + 64} = \sqrt{68} = 2\sqrt{17}$

$CA = \sqrt{(0-5)^2 + (-5-15)^2}$

$= \sqrt{25 + 400} = \sqrt{425} = 5\sqrt{17}$

Here $AB + BC = CA$: $3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$

$\therefore$ The given points are collinear.

(iv) $A(6, 3),\ B(14, 7),\ C(-6, -3)$

$AB = \sqrt{(14-6)^2 + (7-3)^2}$

$= \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$

$BC = \sqrt{(-6-14)^2 + (-3-7)^2}$

$= \sqrt{400 + 100} = \sqrt{500} = 10\sqrt{5}$

$CA = \sqrt{(6-(-6))^2 + (3-(-3))^2}$

$= \sqrt{144 + 36} = \sqrt{180} = 6\sqrt{5}$

Here $AB + CA = BC$: $4\sqrt{5} + 6\sqrt{5} = 10\sqrt{5}$

$\therefore$ The given points are collinear.

(v) $A(0, 15),\ B(2, 9),\ C(7, -6)$

$AB = \sqrt{(2-0)^2 + (9-15)^2}$

$= \sqrt{4 + 36} = \sqrt{40} = 2\sqrt{10}$

$BC = \sqrt{(7-2)^2 + (-6-9)^2}$

$= \sqrt{25 + 225} = \sqrt{250} = 5\sqrt{10}$

$CA = \sqrt{(0-7)^2 + (15-(-6))^2}$

$= \sqrt{49 + 441} = \sqrt{490} = 7\sqrt{10}$

Here $AB + BC = CA$: $2\sqrt{10} + 5\sqrt{10} = 7\sqrt{10}$

$\therefore$ The given points are collinear.

(vi) $A(1, -2),\ B(7, 8),\ C(-2, -7)$

$AB = \sqrt{(7-1)^2 + (8-(-2))^2}$

$= \sqrt{36 + 100} = \sqrt{136} = 2\sqrt{34}$

$BC = \sqrt{(-2-7)^2 + (-7-8)^2}$

$= \sqrt{81 + 225} = \sqrt{306} = 3\sqrt{34}$

$CA = \sqrt{(1-(-2))^2 + (-2-(-7))^2}$

$= \sqrt{9 + 25} = \sqrt{34}$

Here $AB + CA = BC$: $2\sqrt{34} + \sqrt{34} = 3\sqrt{34}$

$\therefore$ The given points are collinear.

Three points are collinear when the sum of the two shorter distances equals the longest distance.

3 **Check whether the given points are the vertices of an isosceles, equilateral, scalene or right triangle. Also find the perimeter.**

(i) $A(2, 3),\ B(5, 3),\ C(2, 1)$

$AB = \sqrt{(5-2)^2 + (3-3)^2}$

$= \sqrt{9 + 0} = \sqrt{9} = 3$

$BC = \sqrt{(2-5)^2 + (1-3)^2}$

$= \sqrt{9 + 4} = \sqrt{13}$

$CA = \sqrt{(2-2)^2 + (3-1)^2}$

$= \sqrt{0 + 4} = \sqrt{4} = 2$

By Pythagoras: $CA^2 + AB^2 = BC^2 \Rightarrow 4 + 9 = 13$

$\therefore$ The points are the vertices of a right (scalene) triangle.

Perimeter $= AB + BC + CA = 3 + \sqrt{13} + 2 = 5 + \sqrt{13}$ units $\approx$ 8.61 units

(ii) $A(0, 0),\ B(1, \sqrt{3}),\ C(2, 0)$

$$AB = \sqrt{(1-0)^2 + (\sqrt{3}-0)^2} = \sqrt{1+3} = 2$$

$$BC = \sqrt{(2-1)^2 + (0-\sqrt{3})^2} = \sqrt{1+3} = 2$$

$$CA = \sqrt{(0-2)^2 + (0-0)^2} = \sqrt{4} = 2$$

All three sides are equal.

$\therefore$ The points are the vertices of an equilateral triangle.

Perimeter $= AB + BC + CA = 2 + 2 + 2 = 6$ units

(iii) $A(0, 1),\ B(8, 4),\ C(0, 8)$

$$AB = \sqrt{(8-0)^2 + (4-1)^2}$$

$$= \sqrt{64+9} = \sqrt{73}$$

$$BC = \sqrt{(0-8)^2 + (8-4)^2}$$

$$= \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$

$$CA = \sqrt{(0-0)^2 + (1-8)^2}$$

$$= \sqrt{0+49} = \sqrt{49} = 7$$

All three sides are different.

$\therefore$ The points are the vertices of a scalene triangle.

Perimeter $= AB + BC + CA = \sqrt{73} + 4\sqrt{5} + 7$ units ≈ 24.49 units

(iv) $A(-3, -5)$, $B(3, -3)$, $C(0, 6)$

$AB = \sqrt{(3-(-3))^2 + (-3-(-5))^2}$

$= \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$

$BC = \sqrt{(0-3)^2 + (6-(-3))^2}$

$= \sqrt{9 + 81} = \sqrt{90} = 3\sqrt{10}$

$CA = \sqrt{(-3-0)^2 + (-5-6)^2}$

$= \sqrt{9 + 121} = \sqrt{130}$

By Pythagoras: $AB^2 + BC^2 = CA^2 \Rightarrow 40 + 90 = 130$

∴ The points are the vertices of a right (scalene) triangle.

Perimeter $= AB + BC + CA = 2\sqrt{10} + 3\sqrt{10} + \sqrt{130} = 5\sqrt{10} + \sqrt{130}$ units ≈ 27.21 units

(v) $A(-1, -1)$, $B(-1, 5)$, $C(4, 2)$

$AB = \sqrt{(-1-(-1))^2 + (5-(-1))^2}$

$= \sqrt{0 + 36} = \sqrt{36} = 6$

$BC = \sqrt{(4-(-1))^2 + (2-5)^2}$

$= \sqrt{25 + 9} = \sqrt{34}$

$CA = \sqrt{(-1-4)^2 + (-1-2)^2}$

$= \sqrt{25 + 9} = \sqrt{34}$

$BC = CA$ (two sides are equal).

∴ The points are the vertices of an isosceles triangle.

Perimeter $= AB + BC + CA = 6 + \sqrt{34} + \sqrt{34} = 6 + 2\sqrt{34}$ units ≈ 17.66 units

(vi) $A(1, 2),\ B(7, 2),\ C(7, 8)$

$AB = \sqrt{(7-1)^2 + (2-2)^2}$

$= \sqrt{36 + 0} = \sqrt{36} = 6$

$BC = \sqrt{(7-7)^2 + (8-2)^2}$

$= \sqrt{0 + 36} = \sqrt{36} = 6$

$CA = \sqrt{(1-7)^2 + (2-8)^2}$

$= \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$

By Pythagoras: $AB^2 + BC^2 = CA^2 \Rightarrow 36 + 36 = 72$

$AB = BC$ (two sides are equal).

$\therefore$ The points are the vertices of a right isosceles triangle.

Perimeter $= AB + BC + CA = 6 + 6 + 6\sqrt{2} = 12 + 6\sqrt{2}$ units

≈ 20.49 units

4 **Show that the points $A(-4, 2)$, $B(1, 4)$, $C(3, -1)$, $D(-2, -3)$ are the vertices of a square.**

$AB = \sqrt{(1-(-4))^2 + (4-2)^2}$

$= \sqrt{25 + 4} = \sqrt{29}$

$DC = \sqrt{(3-(-2))^2 + (-1-(-3))^2}$

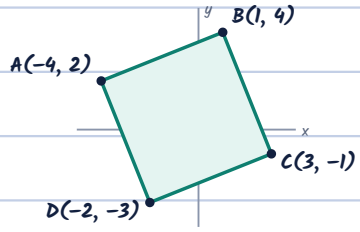
$= \sqrt{25 + 4} = \sqrt{29}$

$BC = \sqrt{(3-1)^2 + (-1-4)^2}$

$= \sqrt{4 + 25} = \sqrt{29}$

$AD = \sqrt{(-2-(-4))^2 + (-3-2)^2}$

$= \sqrt{4 + 25} = \sqrt{29}$



$AB = DC = BC = AD$ (all four sides are equal)

Diagonals:

$AC = \sqrt{(3-(-4))^2 + (-1-2)^2}$

$= \sqrt{49 + 9} = \sqrt{58}$

$BD = \sqrt{(-2-1)^2 + (-3-4)^2}$

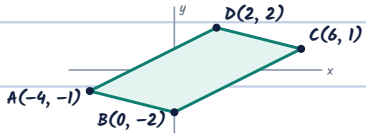
$= \sqrt{9 + 49} = \sqrt{58}$

$AC = BD$ (the diagonals are equal)

All sides are equal and the diagonals are equal.

$\therefore$ $ABCD$ is a square.

5 **Show that the points A(−4, −1), B(0, −2), C(6, 1), D(2, 2) are the vertices of a parallelogram.**



$AB = \sqrt{(0-(-4))^2 + (-2-(-1))^2}$

$= \sqrt{16+1} = \sqrt{17}$

$DC = \sqrt{(6-2)^2 + (1-2)^2}$

$= \sqrt{16+1} = \sqrt{17}$

$BC = \sqrt{(6-0)^2 + (1-(-2))^2}$

$= \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

$AD = \sqrt{(2-(-4))^2 + (2-(-1))^2}$

$= \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

$AB = DC$ and $BC = AD$ (opposite sides are equal)

Diagonals:

$AC = \sqrt{(6-(-4))^2 + (1-(-1))^2}$

$= \sqrt{100+4} = \sqrt{104} = 2\sqrt{26}$

$BD = \sqrt{(2-0)^2 + (2-(-2))^2}$

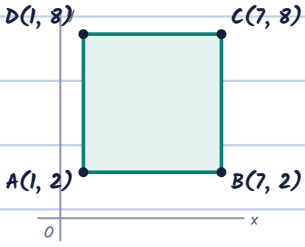
$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$AC \neq BD$ (the diagonals are not equal)

Opposite sides are equal but the diagonals are not equal.

$\therefore$ ABCD is a parallelogram.

6 **Show that the points A(1, 2), B(7, 2), C(7, 8), D(1, 8) are the vertices of a square.**



$AB = \sqrt{(7-1)^2 + (2-2)^2}$

$= \sqrt{36+0} = \sqrt{36} = 6$

$DC = \sqrt{(7-1)^2 + (8-8)^2}$

$= \sqrt{36+0} = \sqrt{36} = 6$

$BC = \sqrt{(7-7)^2 + (8-2)^2}$

$= \sqrt{0+36} = \sqrt{36} = 6$

$AD = \sqrt{(1-1)^2 + (8-2)^2}$

$= \sqrt{0+36} = \sqrt{36} = 6$

$AB = DC = BC = AD$ (all four sides are equal)

Diagonals:

$AC = \sqrt{(7-1)^2 + (8-2)^2}$

$= \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$

$BD = \sqrt{(1-7)^2 + (8-2)^2}$

$= \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$

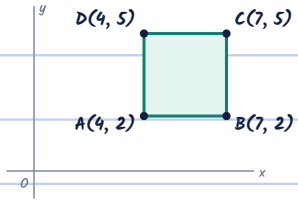
$AC = BD$ (the diagonals are equal)

All sides are equal and the diagonals are equal.

$\therefore$ ABCD is a square.

A square is a special rectangle: its sides are all equal and its angles are right angles.

7 **Show that the points A(4, 2), B(7, 2), C(7, 5), D(4, 5) are the vertices of a square.**



$AB = \sqrt{(7-4)^2 + (2-2)^2}$

$= \sqrt{9 + 0} = \sqrt{9} = 3$

$DC = \sqrt{(7-4)^2 + (5-5)^2}$

$= \sqrt{9 + 0} = \sqrt{9} = 3$

$BC = \sqrt{(7-7)^2 + (5-2)^2}$

$= \sqrt{0 + 9} = \sqrt{9} = 3$

$AD = \sqrt{(4-4)^2 + (5-2)^2}$

$= \sqrt{0 + 9} = \sqrt{9} = 3$

$AB = DC = BC = AD$ (all four sides are equal)

Diagonals:

$AC = \sqrt{(7-4)^2 + (5-2)^2}$

$= \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

$BD = \sqrt{(4-7)^2 + (5-2)^2}$

$= \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

$AC = BD$ (the diagonals are equal)

All sides are equal and the diagonals are equal.

$\therefore$ ABCD is a square.

8 **Show that the points A(−6, 2), B(1, 2), C(1, 5), D(−6, 5) are the vertices of a rectangle.**

$AB = \sqrt{(1-(-6))^2 + (2-2)^2}$

$= \sqrt{49 + 0} = \sqrt{49} = 7$

$DC = \sqrt{(1-(-6))^2 + (5-5)^2}$

$= \sqrt{49 + 0} = \sqrt{49} = 7$

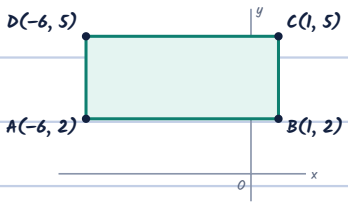
$BC = \sqrt{(1-1)^2 + (5-2)^2}$

$= \sqrt{0 + 9} = \sqrt{9} = 3$

$AD = \sqrt{(-6-(-6))^2 + (5-2)^2}$

$= \sqrt{0 + 9} = \sqrt{9} = 3$

$AB = DC$ and $BC = AD$ (opposite sides are equal)



Diagonals:

$AC = \sqrt{(1-(-6))^2 + (5-2)^2}$

$= \sqrt{49 + 9} = \sqrt{58}$

$BD = \sqrt{(-6-1)^2 + (5-2)^2}$

$= \sqrt{49 + 9} = \sqrt{58}$

$AC = BD$ (the diagonals are equal)

Opposite sides are equal and the diagonals are equal.

∴ ABCD is a rectangle.

9 **Show that the points A(−3, 0), B(3, 0), C(6, 4), D(0, 4) are the vertices of a parallelogram.**

$AB = \sqrt{(3-(-3))^2 + (0-0)^2}$

$= \sqrt{36 + 0} = \sqrt{36} = 6$

$DC = \sqrt{(6-0)^2 + (4-4)^2}$

$= \sqrt{36 + 0} = \sqrt{36} = 6$

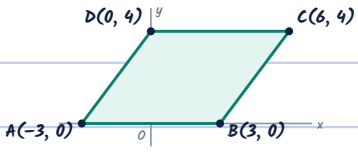
$BC = \sqrt{(6-3)^2 + (4-0)^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$

$AD = \sqrt{(0-(-3))^2 + (4-0)^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$

$AB = DC$ and $BC = AD$ (opposite sides are equal)



Diagonals:

$AC = \sqrt{(6-(-3))^2 + (4-0)^2}$

$= \sqrt{81+16} = \sqrt{97}$

$BD = \sqrt{(0-3)^2 + (4-0)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$AC \neq BD$ (the diagonals are not equal)

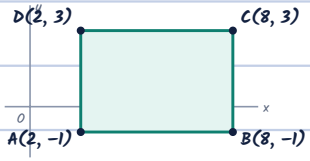
Opposite sides are equal but the diagonals are not equal.

$\therefore$ ABCD is a parallelogram.

10 **Show that the points A(2, −1), B(8, −1), C(8, 3), D(2, 3) are the vertices of a rectangle.**

$AB = \sqrt{(8-2)^2 + (-1-(-1))^2}$

$= \sqrt{36+0} = \sqrt{36} = 6$



$DC = \sqrt{(8-2)^2 + (3-3)^2}$

$= \sqrt{36+0} = \sqrt{36} = 6$

$BC = \sqrt{(8-8)^2 + (3-(-1))^2}$

$= \sqrt{0+16} = \sqrt{16} = 4$

$AD = \sqrt{(2-2)^2 + (3-(-1))^2}$

$= \sqrt{0+16} = \sqrt{16} = 4$

$AB = DC$ and $BC = AD$ (opposite sides are equal)

Diagonals:

$AC = \sqrt{(8-2)^2 + (3-(-1))^2}$

$= \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$BD = \sqrt{(2-8)^2 + (3-(-1))^2}$

$= \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$AC = BD$ (the diagonals are equal)

Opposite sides are equal and the diagonals are equal.

$\therefore$ ABCD is a rectangle.

Also, in $\triangle ABC$: $AC^2 = AB^2 + BC^2 \Rightarrow 52 = 36 + 16$

so $\angle B = 90°$.

In $\triangle ABD$: $BD^2 = AB^2 + AD^2 \Rightarrow 52 = 36 + 16$, so $\angle A = 90°$.

So $\triangle ABC$ and $\triangle ABD$ are right triangles.