Exercise 7.2

Unit 7 · Coordinate Geometry

Midpoint formula: $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

1 **Find the midpoint of the line segment joining each pair of points.**

(i) $(2, 5)$ and $(6, 9)$

$$M\left(\frac{2 + 6}{2}, \frac{5 + 9}{2}\right) = M(4, 7)$$

$\therefore$ Midpoint $= (4, 7)$

(ii) $(-1, 0)$ and $(2, 2)$

$$M\left(\frac{-1 + 2}{2}, \frac{0 + 2}{2}\right) = M\left(\frac{1}{2}, 1\right)$$

$\therefore$ Midpoint $= \left(\frac{1}{2}, 1\right)$

(iii) $(1.4, -1.5)$ and $(2.6, 3.5)$

$$M\left(\frac{1.4 + 2.6}{2}, \frac{-1.5 + 3.5}{2}\right) = M\left(\frac{4}{2}, \frac{2}{2}\right) = M(2, 1)$$

$\therefore$ Midpoint $= (2, 1)$

2 **ABCD is a quadrilateral with A(1, 0), B(5, 2), C(2, 6) and D(−3, 2). E, F, G and H are the midpoints of AB, BC, CD and DA.**

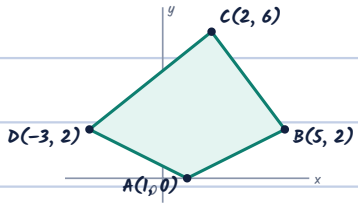
(i) Coordinates of the midpoints:

$$E\left(\frac{1+5}{2}, \frac{0+2}{2}\right) = E(3, 1)$$

$$F\left(\frac{5+2}{2}, \frac{2+6}{2}\right) = F\left(\frac{7}{2}, 4\right)$$

$$G\left(\frac{2+(-3)}{2}, \frac{6+2}{2}\right) = G\left(-\frac{1}{2}, 4\right)$$

$$H\left(\frac{-3+1}{2}, \frac{2+0}{2}\right) = H(-1, 1)$$



(ii) Join the midpoints E, F, G and H to form the quadrilateral EFGH.

(iii) Lengths of the sides of EFGH:

$$EF = \sqrt{\left(\frac{7}{2} - 3\right)^2 + (4 - 1)^2}$$

$$= \sqrt{\frac{1}{4} + 9} = \sqrt{\frac{37}{4}} = \frac{\sqrt{37}}{2}$$

$$FG = \sqrt{\left(-\frac{1}{2} - \frac{7}{2}\right)^2 + (4 - 4)^2}$$

$$= \sqrt{16 + 0} = \sqrt{16} = 4$$

$$GH = \sqrt{\left(-1 - \left(-\frac{1}{2}\right)\right)^2 + (1 - 4)^2}$$

$$= \sqrt{\frac{1}{4} + 9} = \sqrt{\frac{37}{4}} = \frac{\sqrt{37}}{2}$$

$$HE = \sqrt{(3 - (-1))^2 + (1 - 1)^2}$$

$$= \sqrt{16 + 0} = \sqrt{16} = 4$$

(iv) $EF = GH$ and $FG = HE$ (opposite sides are equal)

$\therefore$ EFGH is a parallelogram.

3 **P(1, 0), Q(6, 0), R(7, 4) and S(−1, 4) are the vertices of a trapezium. Do its diagonals bisect each other?**

The diagonals bisect each other only if both diagonals have the same midpoint.

$$\text{Midpoint of } PR = \left(\frac{1+7}{2}, \frac{0+4}{2}\right) = (4, 2)$$

$$\text{Midpoint of } QS = \left(\frac{6+(-1)}{2}, \frac{0+4}{2}\right) = \left(\frac{5}{2}, 2\right)$$

The two midpoints are not the same.

∴ The diagonals of the trapezium do not bisect each other.

4 **A(−3, −1), B(0, 0), C(1, 3) and D(−2, 2) are the vertices of a rhombus. Show that its diagonals bisect each other, and find their lengths.**

$$\text{Midpoint of } AC = \left(\frac{-3+1}{2}, \frac{-1+3}{2}\right) = (-1, 1)$$

$$\text{Midpoint of } BD = \left(\frac{0+(-2)}{2}, \frac{0+2}{2}\right) = (-1, 1)$$

Both diagonals have the same midpoint (−1, 1), so the diagonals bisect each other.

$$AC = \sqrt{(1-(-3))^2 + (3-(-1))^2}$$
$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$
$$BD = \sqrt{(-2-0)^2 + (2-0)^2}$$
$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

∴ $AC = 4\sqrt{2}$ units, $BD = 2\sqrt{2}$ units

5 **A(3, 7), B(−4, 0) and C(4, 0) are the vertices of a triangle. D and E are the midpoints of AB and AC.**

(i) Coordinates of D and E:

$$D\left(\frac{3+(-4)}{2}, \frac{7+0}{2}\right) = D\left(-\frac{1}{2}, \frac{7}{2}\right)$$

$$E\left(\frac{3+4}{2}, \frac{7+0}{2}\right) = E\left(\frac{7}{2}, \frac{7}{2}\right)$$

(ii) Lengths of BC and DE:

$$BC = \sqrt{(4-(-4))^2 + (0-0)^2}$$

$$= \sqrt{64+0} = \sqrt{64} = 8$$

$$DE = \sqrt{\left(\frac{7}{2}-\left(-\frac{1}{2}\right)\right)^2 + \left(\frac{7}{2}-\frac{7}{2}\right)^2}$$

$$= \sqrt{16+0} = \sqrt{16} = 4$$

(iii) $\frac{1}{2}BC = \frac{1}{2} \times 8 = 4$

(iv) Comparing: $DE = 4 = \frac{1}{2}BC$

$\therefore DE = \frac{1}{2}BC$, i.e. $BC = 2\,DE$

The line joining the midpoints of two sides of a triangle is half the third side.

6 **Hassan starts at (6, 0) and Ali starts at (0, 6). They walk towards each other along a straight line and meet at its midpoint C.**

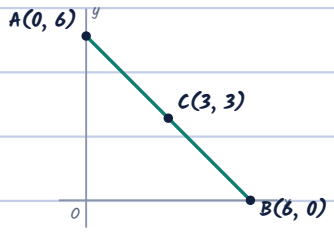
(i) Starting point of Hassan = (6, 0), starting point of Ali = (0, 6)

(ii) Let Ali be at A(0, 6) and Hassan at B(6, 0).

$$C\left(\frac{0+6}{2}, \frac{6+0}{2}\right) = C(3, 3)$$

$$AB = \sqrt{(6-0)^2 + (0-6)^2}$$

$$= \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$$



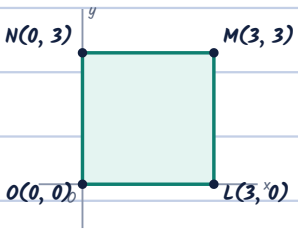
Distance from Ali to C = $\frac{1}{2}$ AB = $\frac{6\sqrt{2}}{2}$ = $3\sqrt{2}$ units

Distance from Hassan to C = $\frac{1}{2}$ AB = $3\sqrt{2}$ units

∴ Each walks $3\sqrt{2}$ units to meet at C(3, 3).

7 **Draw the figures with the given vertices.**

(i) O(0, 0), L(3, 0), M(3, 3), N(0, 3): a square of side 3 units



(ii) A(0, 0), B(0, 2), C(−4, 2), D(−4, 0): a 4 × 2 rectangle

