Exercise 8.2

Unit 8 · Geometry of Straight Lines

Point-slope: $y - y_1 = m(x - x_1)$ Slope-intercept: $y = mx + c$

Two-point: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ Two-intercept: $\frac{x}{a} + \frac{y}{b} = 1$

Symmetric: $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta}$ Normal: $x\cos\theta + y\sin\theta = p$

1 Find the equation of the horizontal line through each point.

A horizontal line has slope $m = 0$, so the point-slope form gives $y - y_1 = 0$.

(i) $(2, 3)$

$y - 3 = 0(x - 2)$

$\therefore y - 3 = 0$

(ii) $(8, 0)$

$y - 0 = 0(x - 8)$

$\therefore y = 0$

(iii) $(-5, -9)$

$y - (-9) = 0(x + 5)$

$\therefore y + 9 = 0$

(iv) $(-\frac{3}{2}, -\frac{5}{2})$

$y + \frac{5}{2} = 0$

$\therefore 2y + 5 = 0$

2 Find the equation of the vertical line through each point.

A vertical line has an undefined slope. Every point on it has the same x-coordinate, so its equation is $x = x_1$, i.e. $x - x_1 = 0$.

(i) $(1, 5)$

$x = 1$

$\therefore x - 1 = 0$

(ii) $(9, 6)$

$x = 9$

$\therefore x - 9 = 0$

(iii) $(-4, -7)$

$x = -4$

$\therefore x + 4 = 0$

(iv) $\left(\frac{1}{4}, \frac{3}{4}\right)$

$x = \frac{1}{4}$

$\therefore 4x - 1 = 0$

In the notes this is also shown as $m = 1/0$ in the point-slope form, which gives the same answer.

3 **Find the equation of the line in each case.**

(i) *Slope = 2, y-intercept = −3*

$m = 2$, $c = -3$. Slope-intercept form: $y = mx + c$

$y = 2x - 3$

$\therefore 2x - y - 3 = 0$

(ii) *Through (−5, 7) with slope 4*

$y - y_1 = m(x - x_1)$

$y - 7 = 4(x + 5) \Rightarrow y - 7 = 4x + 20$

$\therefore 4x - y + 27 = 0$

(iii) *Through (4, −5) with slope 0*

$y - (-5) = 0(x - 4)$

$\therefore y + 5 = 0$

(iv) *Through (−2, 9) with undefined slope*

The line is vertical, so $x = -2$.

$\therefore x + 2 = 0$

(v) *Through (−6, 1) and (2, −4)*

$$y - 1 = \frac{-4 - 1}{2 - (-6)}(x + 6) = \frac{-5}{8}(x + 6)$$

$8y - 8 = -5x - 30 \Rightarrow 5x + 8y = -22$

$\therefore 5x + 8y + 22 = 0$

(vi) Through $(2, -4)$ and $(8, 4)$

$$y + 4 = \frac{4 - (-4)}{8 - 2}(x - 2) = \frac{4}{3}(x - 2)$$

$$3y + 12 = 4x - 8$$

$$\therefore 4x - 3y - 20 = 0$$

(vii) x-intercept $= -6$, y-intercept $= 5$

$$a = -6,\ b = 5:\ \frac{x}{-6} + \frac{y}{5} = 1$$

Multiply by 30: $-5x + 6y = 30$

$$\therefore 5x - 6y + 30 = 0$$

(viii) Slope $= -1$, x-intercept $= 11$

The line passes through $(11, 0)$.

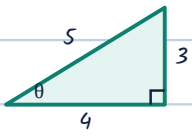
$$y - 0 = -1(x - 11) \Rightarrow y = -x + 11$$

$$\therefore x + y - 11 = 0$$

4 **Find the equation of the line through the given point using the symmetric form.**

(i) Through $(-4, 2)$ with $\tan\theta = \frac{3}{4}$

$\tan\theta = \frac{3}{4} \Rightarrow$ perpendicular $= 3$, base $= 4$



Hypotenuse $= \sqrt{9 + 16} = \sqrt{25} = 5$

$\sin\theta = \frac{3}{5}, \quad \cos\theta = \frac{4}{5}$

$$\frac{x + 4}{\cos\theta} = \frac{y - 2}{\sin\theta}$$

$$\Rightarrow \frac{x + 4}{\frac{4}{5}} = \frac{y - 2}{\frac{3}{5}}$$

$$\Rightarrow \frac{x + 4}{4} = \frac{y - 2}{3}$$

$$3x + 12 = 4y - 8$$

$$\therefore 3x - 4y + 20 = 0$$

(ii) Through $(6, -6)$ with $\theta = 30°$

$$\frac{x - 6}{\cos 30°} = \frac{y + 6}{\sin 30°}$$

$$\Rightarrow \frac{x - 6}{\frac{\sqrt{3}}{2}} = \frac{y + 6}{\frac{1}{2}}$$

$$x - 6 = \sqrt{3}(y + 6)$$

$$\Rightarrow x - 6 = \sqrt{3}\,y + 6\sqrt{3}$$

$$\therefore x - \sqrt{3}\,y - 6 - 6\sqrt{3} = 0$$

5 **Find the equation of the line in normal form, $x \cos\theta + y \sin\theta = p$.**

(i) $p = 5, \quad \theta = 120°$

$x \cos 120° + y \sin 120° = 5$

$\Rightarrow \quad x\left(-\frac{1}{2}\right) + y\left(\frac{\sqrt{3}}{2}\right) = 5$

$-x + \sqrt{3}\,y = 10$

$\therefore\ x - \sqrt{3}\,y + 10 = 0$

(ii) $p = 10, \quad \tan\theta = 1$

$\tan\theta = 1 \quad \Rightarrow \quad \theta = 45°$

$\text{so } \sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$

$x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right) = 10$

$\Rightarrow \quad x + y = 10\sqrt{2}$

$\therefore\ x + y - 10\sqrt{2} = 0$

6 **Find the equation of the line in each case.**

(i) Through $(-4, -4)$, parallel to a line with slope -5

Parallel lines have equal slopes: $m = -5$

$y + 4 = -5(x + 4) \quad \Rightarrow \quad y + 4 = -5x - 20$

$\therefore\ 5x + y + 24 = 0$

(ii) Through $(5, -1)$, perpendicular to a line with slope $\frac{1}{4}$

$$m = \frac{-1}{\frac{1}{4}} = -4$$

$$y + 1 = -4(x - 5) \Rightarrow y + 1 = -4x + 20$$

$$\therefore 4x + y - 19 = 0$$

(iii) y-intercept 4, parallel to a line with slope $\frac{1}{2}$

$$m = \frac{1}{2},\ c = 4:\ y = \frac{1}{2}x + 4 \Rightarrow 2y = x + 8$$

$$\therefore x - 2y + 8 = 0$$

(iv) x-intercept -2, perpendicular to a line with slope 4

Point $(-2, 0)$, $m = -\frac{1}{4}$

$$y - 0 = -\frac{1}{4}(x + 2) \Rightarrow 4y = -x - 2$$

$$\therefore x + 4y + 2 = 0$$

(v) Through $(-1, 4)$, perpendicular to the line through $(3, 0)$ and $(1, -2)$

$$\text{Slope of that line} = \frac{-2 - 0}{1 - 3} = \frac{-2}{-2} = 1$$

$$\text{so } m = \frac{-1}{1} = -1$$

$$y - 4 = -1(x + 1) \Rightarrow y - 4 = -x - 1$$

$$\therefore x + y - 3 = 0$$

(vi) Through $(6, -4)$, parallel to the line through $(-5, 2)$ and $(3, 6)$

$$\text{Slope} = \frac{6-2}{3+5} = \frac{4}{8} = \frac{1}{2}$$

$$y + 4 = \frac{1}{2}(x - 6) \Rightarrow 2y + 8 = x - 6$$

$$\therefore x - 2y - 14 = 0$$

7 **Find the equation of the line through (3, 7) parallel to $4x - 3y + 1 = 0$.**

$$\text{Slope of } 4x - 3y + 1 = 0 \text{ is } m = \frac{-a}{b} = \frac{-4}{-3} = \frac{4}{3}$$

Parallel line: $m = \frac{4}{3}$. $y - 7 = \frac{4}{3}(x - 3)$

$$\Rightarrow 3y - 21 = 4x - 12$$

$$\therefore 4x - 3y + 9 = 0$$

8 **Find the equation of the line through $(-2, -1)$ perpendicular to $x - 2y = 0$.**

$$\text{Slope of } x - 2y = 0 \text{ is } \frac{-a}{b} = \frac{-1}{-2} = \frac{1}{2}$$

$$\text{so } m = \frac{-1}{\frac{1}{2}} = -2$$

$$y + 1 = -2(x + 2) \Rightarrow y + 1 = -2x - 4$$

$$\therefore 2x + y + 5 = 0$$

9 **Find the equation of the perpendicular bisector of the line segment joining A(0, 6) and B(2, −2).**

$$\text{Midpoint of } AB = \left(\frac{0+2}{2}, \frac{6-2}{2}\right) = (1, 2)$$

$$\text{Slope of } AB = \frac{-2-6}{2-0} = -4$$

$$\text{Slope of the bisector} = \frac{-1}{-4} = \frac{1}{4}$$

$$y - 2 = \frac{1}{4}(x - 1) \Rightarrow 4y - 8 = x - 1$$

$$\therefore x - 4y + 7 = 0$$

10 **A(0, 4), B(4, 6) and C(−2, −2) are the vertices of a triangle. Find the equations of its medians and altitudes.**

Medians. Let M_1, M_2, M_3 be the midpoints of BC, CA and AB.

$$M_1\left(\frac{4-2}{2}, \frac{6-2}{2}\right) = M_1(1, 2)$$

$$M_2\left(\frac{0-2}{2}, \frac{4-2}{2}\right) = M_2(-1, 1)$$

$$M_3\left(\frac{0+4}{2}, \frac{4+6}{2}\right) = M_3(2, 5)$$

Median AM_1: $y - 4 = \frac{2-4}{1-0}(x-0) = -2x$

$\therefore\ 2x + y - 4 = 0$

Median BM_2: $y - 6 = \frac{1-6}{-1-4}(x-4) = 1(x-4)$

$\therefore\ x - y + 2 = 0$

Median CM_3: $y + 2 = \frac{5+2}{2+2}(x+2)$

$\Rightarrow\ 4y + 8 = 7x + 14$

$\therefore\ 7x - 4y + 6 = 0$

Altitudes. *Each altitude goes through a vertex and is perpendicular to the opposite side.*

Slope of $AB = \frac{6-4}{4-0} = \frac{1}{2}$

Slope of $BC = \frac{6+2}{4+2} = \frac{4}{3}$

Slope of $AC = \frac{4+2}{0+2} = 3$

Altitude from C ($\perp$ AB): $m = -2$, $y + 2 = -2(x+2)$

$\therefore\ 2x + y + 6 = 0$

Altitude from A ($\perp$ BC): $m = -\frac{3}{4}$

$y - 4 = -\frac{3}{4}x \Rightarrow 4y - 16 = -3x$

$\therefore\ 3x + 4y - 16 = 0$

Altitude from B ($\perp$ AC): $m = -\frac{1}{3}$

$y - 6 = -\frac{1}{3}(x-4) \Rightarrow 3y - 18 = -x + 4$

$\therefore\ x + 3y - 22 = 0$