Exercise 8.3

Unit 8 · Geometry of Straight Lines

Angle θ from l_1 to l_2: $\tan\theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$ ($\theta = 90°$ if $1 + m_1 m_2 = 0$)

Intersection of $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$:

$$(x, y) = \left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \right)$$

Slope of $ax + by + c = 0$ is $m = -a/b$.

Family of lines through the intersection of l_1 and l_2: $l_1 + k \cdot l_2 = 0$

1 **Find the angle θ from l_1 to l_2 when their slopes are given.**

(i) $m_1 = 0, \quad m_2 = 1$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{1 - 0}{1 + (0)(1)}$$

$$\tan\theta = 1 \Rightarrow \theta = \tan^{-1}(1) = 45°$$

$\therefore \theta = 45°$

(ii) $m_1 = -0.5, \quad m_2 = 4.5$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{4.5 - (-0.5)}{1 + (-0.5)(4.5)}$$

$$\tan\theta = -4 \Rightarrow \tan^{-1}(-4) = -75.96°$$

$$\Rightarrow \theta = 180° - 75.96° = 104.04°$$

$\therefore \theta = 104.04°$

(iii) $m_1 = \tan 45° = 1, \quad m_2 = \tan 135° = -1$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{-1 - 1}{1 + (1)(-1)}$$

The denominator is 0, so $\tan\theta$ is undefined $\Rightarrow \theta = 90°$

$\therefore \theta = 90°$

2 **Find the angle θ from l_1 to l_2, where each line joins the given points.**

(i) l_1 joins (2, 0) and (5, 0); l_2 joins (2, 0) and (5, 5)

$$m_1 = \frac{0 - 0}{5 - 2} = 0$$

$$m_2 = \frac{5 - 0}{5 - 2} = \frac{5}{3}$$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{\frac{5}{3} - 0}{1 + (0)(\frac{5}{3})}$$

$$\tan\theta = \frac{5}{3} \Rightarrow \theta = \tan^{-1}(\frac{5}{3}) = 59.04°$$

$\therefore \theta = 59.04°$

(ii) l_1 joins $(-2, 1)$ and $(3, 4)$; l_2 joins $(-1, 3)$ and $(4, 8)$

$$m_1 = \frac{4 - 1}{3 - (-2)} = \frac{3}{5}$$

$$m_2 = \frac{8 - 3}{4 - (-1)} = 1$$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{1 - \frac{3}{5}}{1 + (\frac{3}{5})(1)}$$

$$\tan\theta = \frac{1}{4} \Rightarrow \theta = \tan^{-1}(\frac{1}{4}) = 14.04°$$

$\therefore \theta = 14.04°$

(iii) l_1 joins $(-5, -4)$ and $(5, 1)$; l_2 joins $(-3, 2)$ and $(0, 5)$

$$m_1 = \frac{1 - (-4)}{5 - (-5)} = \frac{1}{2}$$

$$m_2 = \frac{5 - 2}{0 - (-3)} = 1$$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{1 - \frac{1}{2}}{1 + (\frac{1}{2})(1)}$$

$$\tan\theta = \frac{1}{3} \Rightarrow \theta = \tan^{-1}(\frac{1}{3}) = 18.43°$$

$\therefore \theta = 18.43°$

(iv) l_1 joins $(2, -6)$ and $(5, -9)$; l_2 joins $(5, -5)$ and $(-10, -5)$

$$m_1 = \frac{-9 - (-6)}{5 - 2} = -1$$

$$m_2 = \frac{-5 - (-5)}{-10 - 5} = 0$$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{0 - (-1)}{1 + (-1)(0)}$$

$$\tan\theta = 1 \Rightarrow \theta = \tan^{-1}(1) = 45°$$

$\therefore \theta = 45°$

(v) l_1 joins $(0, -3)$ and $(7, -9)$; l_2 joins $(2, -2)$ and $(-8, -12)$

$$m_1 = \frac{-9 - (-3)}{7 - 0} = -\frac{6}{7}$$

$$m_2 = \frac{-12 - (-2)}{-8 - 2} = 1$$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{1 - (-\frac{6}{7})}{1 + (-\frac{6}{7})(1)}$$

$$\tan\theta = 13 \Rightarrow \theta = \tan^{-1}(13) = 85.6°$$

$\therefore \theta = 85.60°$

3 **Find the interior angles of triangle ABC from the slopes of its sides.**

Let m_1 = slope of AB, m_2 = slope of BC, m_3 = slope of AC.

$$\tan A = \frac{m_3 - m_1}{1 + m_1 m_3}, \quad \tan B = \frac{m_1 - m_2}{1 + m_1 m_2}, \quad \tan C = \frac{m_2 - m_3}{1 + m_2 m_3}$$

(i) $m_1 = 0,\ m_2 = -1,\ m_3 = 1$

$$\tan A = \frac{m_3 - m_1}{1 + m_1 m_3} = \frac{1 - 0}{1 + (0)(1)}$$

$$\tan A = 1 \Rightarrow A = \tan^{-1}(1) = 45°$$

$$\tan B = \frac{m_1 - m_2}{1 + m_2 m_1} = \frac{0 - (-1)}{1 + (-1)(0)}$$

$$\tan B = 1 \Rightarrow B = \tan^{-1}(1) = 45°$$

$$\tan C = \frac{m_2 - m_3}{1 + m_3 m_2} = \frac{-1 - 1}{1 + (1)(-1)}$$

The denominator is 0, so tan C is undefined $\Rightarrow$ $C = 90°$

$\therefore \angle A = 45°,\ \angle B = 45°,\ \angle C = 90°$

Check: $45° + 45° + 90° = 180°$

(ii) $m_1 = 0.25,\ m_2 = 1.25,\ m_3 = 1$

$$\tan A = \frac{m_3 - m_1}{1 + m_1 m_3} = \frac{1 - 0.25}{1 + (0.25)(1)}$$

$$\tan A = 0.6 \Rightarrow A = \tan^{-1}(0.6) = 30.96°$$

$$\tan B = \frac{m_1 - m_2}{1 + m_2 m_1} = \frac{0.25 - 1.25}{1 + (1.25)(0.25)}$$

$$\tan B = -0.7619 \Rightarrow \tan^{-1}(-0.7619) = -37.3°$$

$$\Rightarrow B = 180° - 37.3° = 142.7°$$

$$\tan C = \frac{m_2 - m_3}{1 + m_3 m_2} = \frac{1.25 - 1}{1 + (1)(1.25)}$$

$$\tan C = 0.1111 \Rightarrow C = \tan^{-1}(0.1111) = 6.34°$$

$\therefore \angle A = 30.96°, \quad \angle B = 142.70°, \quad \angle C = 6.34°$

Check: $30.96° + 142.70° + 6.34° = 180°$

(iii) $m_1 = 0.4, \quad m_2 = -1.5, \quad m_3 = 1.667$

$$\tan A = \frac{m_3 - m_1}{1 + m_1 m_3} = \frac{1.667 - 0.4}{1 + (0.4)(1.667)}$$

$$\tan A = 0.7601 \Rightarrow A = \tan^{-1}(0.7601) = 37.24°$$

$$\tan B = \frac{m_1 - m_2}{1 + m_2 m_1} = \frac{0.4 - (-1.5)}{1 + (-1.5)(0.4)}$$

$$\tan B = 4.75 \Rightarrow B = \tan^{-1}(4.75) = 78.11°$$

$$\tan C = \frac{m_2 - m_3}{1 + m_3 m_2} = \frac{-1.5 - 1.667}{1 + (1.667)(-1.5)}$$

$$\tan C = 2.1106 \Rightarrow C = \tan^{-1}(2.1106) = 64.65°$$

$\therefore \angle A = 37.24°, \quad \angle B = 78.11°, \quad \angle C = 64.65°$

Check: $37.24° + 78.11° + 64.65° = 180°$

(iv) $m_1 = -1, \quad m_2 = 0.8, \quad m_3 = 0$

$$\tan A = \frac{m_3 - m_1}{1 + m_1 m_3} = \frac{0 - (-1)}{1 + (-1)(0)}$$

$$\tan A = 1 \Rightarrow A = \tan^{-1}(1) = 45°$$

$$\tan B = \frac{m_1 - m_2}{1 + m_2 m_1} = \frac{-1 - 0.8}{1 + (0.8)(-1)}$$

$$\tan B = -9 \Rightarrow \tan^{-1}(-9) = -83.66°$$

$$\Rightarrow B = 180° - 83.66° = 96.34°$$

$$\tan C = \frac{m_2 - m_3}{1 + m_3 m_2} = \frac{0.8 - 0}{1 + (0)(0.8)}$$

$$\tan C = 0.8 \Rightarrow C = \tan^{-1}(0.8) = 38.66°$$

$\therefore \angle A = 45°, \ \angle B = 96.34°, \ \angle C = 38.66°$

Check: $45° + 96.34° + 38.66° = 180°$

4 **Find the angle θ from the first line to the second line.**

(i) $3x + 2y + 5 = 0$ and $2x - 3y + 8 = 0$

$$m_1 = -\frac{3}{2}, \quad m_2 = \frac{2}{3} \quad (m = -a/b)$$

$$\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{\frac{2}{3} - (-\frac{3}{2})}{1 + (-\frac{3}{2})(\frac{2}{3})}$$

The denominator is 0, so $\tan \theta$ is undefined $\Rightarrow \theta = 90°$

$\therefore \theta = 90°$

(ii) $x + 2y - 6 = 0$ and $2x - 4y + 9 = 0$

$$m_1 = -\frac{1}{2}, \quad m_2 = \frac{1}{2} \quad (m = -a/b)$$

$$\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{\frac{1}{2} - (-\frac{1}{2})}{1 + (-\frac{1}{2})(\frac{1}{2})}$$

$$\tan \theta = \frac{4}{3} \Rightarrow \theta = \tan^{-1}(\frac{4}{3}) = 53.13°$$

$\therefore \theta = 53.13°$

(iii) $6x - y + 1 = 0$ and $x - 7y + 12 = 0$

$m_1 = 6, \quad m_2 = \frac{1}{7} \quad (m = -a/b)$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{\frac{1}{7} - 6}{1 + (6)(\frac{1}{7})}$$

$$\tan\theta = -\frac{41}{13} \Rightarrow \tan^{-1}(-\frac{41}{13}) = -72.41°$$

$\Rightarrow \theta = 180° - 72.41° = 107.59°$

$\therefore \theta = 107.59°$

5 **Find the interior angles α (at X), β (at Y) and γ (at Z) of triangle XYZ.**

(i) $X(-2, 3), \quad Y(-3, -4), \quad Z(5, 2)$

$$m_1 = \text{slope of } XY = \frac{-4 - 3}{-3 - (-2)} = 7$$

$$m_2 = \text{slope of } YZ = \frac{2 - (-4)}{5 - (-3)} = \frac{3}{4}$$

$$m_3 = \text{slope of } XZ = \frac{2 - 3}{5 - (-2)} = -\frac{1}{7}$$

$$\tan\alpha = \frac{m_3 - m_1}{1 + m_1 m_3} = \frac{-\frac{1}{7} - 7}{1 + (7)(-\frac{1}{7})}$$

The denominator is 0, so $\tan\alpha$ is undefined $\Rightarrow \alpha = 90°$

$$\tan\beta = \frac{m_1 - m_2}{1 + m_2 m_1} = \frac{7 - \frac{3}{4}}{1 + (\frac{3}{4})(7)}$$

$\tan\beta = 1 \Rightarrow \beta = \tan^{-1}(1) = 45°$

$$\tan\gamma = \frac{m_2 - m_3}{1 + m_3 m_2} = \frac{\frac{3}{4} - (-\frac{1}{7})}{1 + (-\frac{1}{7})(\frac{3}{4})}$$

$$\tan\gamma = 1 \Rightarrow \gamma = \tan^{-1}(1) = 45°$$

$\therefore \alpha = 90°,\ \beta = 45°,\ \gamma = 45°$

(ii) $X(-3, 2),\ Y(0, -1),\ Z(3, 3)$

$$m_1 = \text{slope of } XY = \frac{-1 - 2}{0 - (-3)} = -1$$

$$m_2 = \text{slope of } YZ = \frac{3 - (-1)}{3 - 0} = \frac{4}{3}$$

$$m_3 = \text{slope of } XZ = \frac{3 - 2}{3 - (-3)} = \frac{1}{6}$$

$$\tan\alpha = \frac{m_3 - m_1}{1 + m_1 m_3} = \frac{\frac{1}{6} - (-1)}{1 + (-1)(\frac{1}{6})}$$

$$\tan\alpha = \frac{7}{5} \Rightarrow \alpha = \tan^{-1}(\frac{7}{5}) = 54.46°$$

$$\tan\beta = \frac{m_1 - m_2}{1 + m_2 m_1} = \frac{-1 - \frac{4}{3}}{1 + (\frac{4}{3})(-1)}$$

$$\tan\beta = 7 \Rightarrow \beta = \tan^{-1}(7) = 81.87°$$

$$\tan\gamma = \frac{m_2 - m_3}{1 + m_3 m_2} = \frac{\frac{4}{3} - \frac{1}{6}}{1 + (\frac{1}{6})(\frac{4}{3})}$$

$$\tan\gamma = \frac{21}{22} \Rightarrow \gamma = \tan^{-1}(\frac{21}{22}) = 43.67°$$

$\therefore \alpha = 54.46°,\ \beta = 81.87°,\ \gamma = 43.67°$

(iii) $X(-2, 0),\ Y(1, -4),\ Z(6, 6)$

$$m_1 = \text{slope of } XY = \frac{-4 - 0}{1 - (-2)} = -\frac{4}{3}$$

$$m_2 = \text{slope of } YZ = \frac{6 - (-4)}{6 - 1} = 2$$

$$m_3 = \text{slope of } XZ = \frac{6 - 0}{6 - (-2)} = \frac{3}{4}$$

$$\tan\alpha = \frac{m_3 - m_1}{1 + m_1 m_3} = \frac{\frac{3}{4} - (-\frac{4}{3})}{1 + (-\frac{4}{3})(\frac{3}{4})}$$

The denominator is 0, so $\tan\alpha$ is undefined $\Rightarrow$ $\alpha = 90°$

$$\tan\beta = \frac{m_1 - m_2}{1 + m_2 m_1} = \frac{-\frac{4}{3} - 2}{1 + (2)(-\frac{4}{3})}$$

$$\tan\beta = 2 \Rightarrow \beta = \tan^{-1}(2) = 63.43°$$

$$\tan\gamma = \frac{m_2 - m_3}{1 + m_3 m_2} = \frac{2 - \frac{3}{4}}{1 + (\frac{3}{4})(2)}$$

$$\tan\gamma = \frac{1}{2} \Rightarrow \gamma = \tan^{-1}(\frac{1}{2}) = 26.57°$$

$\therefore \alpha = 90°,\ \beta = 63.43°,\ \gamma = 26.57°$

(iv) $X(-4, 1),\ Y(0, -3),\ Z(4, 3)$

$$m_1 = \text{slope of } XY = \frac{-3 - 1}{0 - (-4)} = -1$$

$$m_2 = \text{slope of } YZ = \frac{3 - (-3)}{4 - 0} = \frac{3}{2}$$

$$m_3 = \text{slope of } XZ = \frac{3 - 1}{4 - (-4)} = \frac{1}{4}$$

$$\tan\alpha = \frac{m_3 - m_1}{1 + m_1 m_3} = \frac{\frac{1}{4} - (-1)}{1 + (-1)(\frac{1}{4})}$$

$$\tan\alpha = \frac{5}{3} \Rightarrow \alpha = \tan^{-1}(\frac{5}{3}) = 59.04°$$

$$\tan\beta = \frac{m_1 - m_2}{1 + m_2 m_1} = \frac{-1 - \frac{3}{2}}{1 + (\frac{3}{2})(-1)}$$

$$\tan\beta = 5 \Rightarrow \beta = \tan^{-1}(5) = 78.69°$$

$$\tan\gamma = \frac{m_2 - m_3}{1 + m_3 m_2} = \frac{\frac{3}{2} - \frac{1}{4}}{1 + (\frac{1}{4})(\frac{3}{2})}$$

$$\tan\gamma = \frac{10}{11} \Rightarrow \gamma = \tan^{-1}(\frac{10}{11}) = 42.27°$$

$\therefore \alpha = 59.04°, \ \beta = 78.69°, \ \gamma = 42.27°$

6 Find the point of intersection of the lines.

(i) $2x + y + 1 = 0$ and $x - y - 4 = 0$

$a_1 = 2, b_1 = 1, c_1 = 1; \quad a_2 = 1, b_2 = (-1), c_2 = (-4)$

$$x = \frac{1(-4) - (-1)1}{2(-1) - 11} = \frac{-3}{-3} = 1$$

$$y = \frac{11 - 2(-4)}{2(-1) - 11} = \frac{9}{-3} = -3$$

$\therefore$ Point of intersection $= (1, -3)$

(ii) $x + y + 3 = 0$ and $2x - 5y + 8 = 0$

$a_1 = 1, b_1 = 1, c_1 = 3;\quad a_2 = 2, b_2 = (-5), c_2 = 8$

$$x = \frac{18 - (-5)3}{1(-5) - 21} = \frac{23}{-7} = -\frac{23}{7}$$

$$y = \frac{23 - 18}{1(-5) - 21} = \frac{-2}{-7} = \frac{2}{7}$$

$\therefore$ Point of intersection $= (-\frac{23}{7}, \frac{2}{7})$

(iii) $2x + 5y + 3 = 0$ and $3x - 4y - 5 = 0$

$a_1 = 2, b_1 = 5, c_1 = 3;\quad a_2 = 3, b_2 = (-4), c_2 = (-5)$

$$x = \frac{5(-5) - (-4)3}{2(-4) - 35} = \frac{-13}{-23} = \frac{13}{23}$$

$$y = \frac{33 - 2(-5)}{2(-4) - 35} = \frac{19}{-23} = -\frac{19}{23}$$

$\therefore$ Point of intersection $= (\frac{13}{23}, -\frac{19}{23})$

7 **Using l_1: $3x + 2y + 1 = 0$ and l_2: $x - 2y + 3 = 0$, find the line through their intersection that (a) passes through $(-1, 0)$, (b) is parallel to $3x - 4y + 3 = 0$.**

Family: $3x + 2y + 1 + k(x - 2y + 3) = 0$

$(k + 3)x + (2 - 2k)y + (1 + 3k) = 0$ ...(iii)

(a) Through $(-1, 0)$: $(k + 3)(-1) + 0 + 1 + 3k = 0 \Rightarrow 2k - 2 = 0 \Rightarrow k = 1$

Put $k = 1$ in (iii): $4x + 0y + 4 = 0 \Rightarrow 4(x + 1) = 0$

$\therefore x + 1 = 0$

(b) Parallel to $3x - 4y + 3 = 0$

Slope of (iii) $= \dfrac{-(k+3)}{2-2k}$

Slope of $3x - 4y + 3 = 0 = \dfrac{3}{4}$

Parallel: $\dfrac{-(k+3)}{2-2k} = \dfrac{3}{4}$

$\Rightarrow \quad -4k - 12 = 6 - 6k \quad \Rightarrow \quad 2k = 18 \quad \Rightarrow \quad k = 9$

Put $k = 9$ in (iii): $12x - 16y + 28 = 0$

$\Rightarrow \quad 4(3x - 4y + 7) = 0$

$\therefore 3x - 4y + 7 = 0$

8 **Find the line through the intersection of $6x + 5y + 3 = 0$ and $2x - 5y + 13 = 0$ with slope 3.**

Family: $6x + 5y + 3 + k(2x - 5y + 13) = 0$

$\Rightarrow \quad (6 + 2k)x + (5 - 5k)y + (3 + 13k) = 0$

Slope $= \dfrac{-(6+2k)}{5-5k} = 3 \quad \Rightarrow \quad -6 - 2k = 15 - 15k$

$\Rightarrow \quad 13k = 21 \quad \Rightarrow \quad k = \dfrac{21}{13}$

$13(6x + 5y + 3) + 21(2x - 5y + 13) = 0$

$\Rightarrow \quad 120x - 40y + 312 = 0$

$\therefore 15x - 5y + 39 = 0$

9 **Find the line through the intersection of $2x - 5y + 4 = 0$ and $6x - 4y + 5 = 0$ that is (a) parallel to the x-axis, (b) parallel to the y-axis.**

Family: $2x - 5y + 4 + k(6x - 4y + 5) = 0$

$\Rightarrow (2 + 6k)x + (-5 - 4k)y + (4 + 5k) = 0 \quad ...(i)$

(a) Parallel to the x-axis: the x-term must vanish, so $2 + 6k = 0$

$\Rightarrow k = -\frac{1}{3}$

$3(2x - 5y + 4) - (6x - 4y + 5) = 0 \Rightarrow -11y + 7 = 0$

$\therefore 11y - 7 = 0$

(b) Parallel to the y-axis: the y-term must vanish, so $-5 - 4k = 0$

$\Rightarrow k = -\frac{5}{4}$

$4(2x - 5y + 4) - 5(6x - 4y + 5) = 0 \Rightarrow -22x - 9 = 0$

$\therefore 22x + 9 = 0$

10 **Find the line through the intersection of $2x - y + 2 = 0$ and $x - 2y + 1 = 0$ that is (a) parallel to $x - 2y + 11 = 0$, (b) perpendicular to $2x + 5y + 2 = 0$.**

Family: $2x - y + 2 + k(x - 2y + 1) = 0$

$\Rightarrow (2 + k)x + (-1 - 2k)y + (2 + k) = 0 \quad ...(i)$

(a) Slope of $x - 2y + 11 = 0$ is $\frac{1}{2}$.

Family method: $\frac{k + 2}{1 + 2k} = \frac{1}{2}$

$\Rightarrow \quad 2k + 4 = 2k + 1$

This is impossible.

No value of k works, because the required line is the second given line itself ($x - 2y + 1 = 0$), which the family form only gives when k is infinitely large. So use another method:

Solve $2x - y + 2 = 0$ and $x - 2y + 1 = 0$:

$2(2x - y + 2) - (x - 2y + 1) = 0 \quad \Rightarrow \quad 3x + 3 = 0$

$x = -1, \quad y = 0$

Line through $(-1, 0)$ with slope $\frac{1}{2}$: $y - 0 = \frac{1}{2}(x + 1)$

$\Rightarrow \quad 2y = x + 1$

$\therefore x - 2y + 1 = 0$

(b) Slope of $2x + 5y + 2 = 0$ is $-\frac{2}{5}$.

Perpendicular: $\frac{k + 2}{1 + 2k} \times \left(-\frac{2}{5}\right) = -1$

$-2k - 4 = -5 - 10k \quad \Rightarrow \quad 8k = -1 \quad \Rightarrow \quad k = -\frac{1}{8}$

$8(2x - y + 2) - (x - 2y + 1) = 0 \quad \Rightarrow \quad 15x - 6y + 15 = 0$

$\Rightarrow \quad 3(5x - 2y + 5) = 0$

$\therefore 5x - 2y + 5 = 0$

11 **Find the line through the intersection of $x - 2y + 4 = 0$ and $3x - y - 3 = 0$ that is (a) parallel, (b) perpendicular to the line through $(2, -3)$ and $(0, 4)$.**

Family: $x - 2y + 4 + k(3x - y - 3) = 0$

$\Rightarrow (1 + 3k)x + (-2 - k)y + (4 - 3k) = 0$, slope $= \dfrac{1 + 3k}{2 + k}$

Slope of the line through $(2, -3)$ and $(0, 4) = \dfrac{4 - (-3)}{0 - 2} = -\dfrac{7}{2}$

(a) Parallel

$$\frac{1 + 3k}{2 + k} = -\frac{7}{2} \Rightarrow 2 + 6k = -14 - 7k$$

$$13k = -16 \Rightarrow k = -\frac{16}{13}$$

$$13(x - 2y + 4) - 16(3x - y - 3) = 0$$

$$\Rightarrow -35x - 10y + 100 = 0 \Rightarrow -5(7x + 2y - 20) = 0$$

$\therefore 7x + 2y - 20 = 0$

(b) Perpendicular

$$\frac{1 + 3k}{2 + k} \times \left(-\frac{7}{2}\right) = -1$$

$$\Rightarrow 7 + 21k = 4 + 2k \Rightarrow k = -\frac{3}{19}$$

$$19(x - 2y + 4) - 3(3x - y - 3) = 0$$

$$\Rightarrow 10x - 35y + 85 = 0 \Rightarrow 5(2x - 7y + 17) = 0$$

$\therefore 2x - 7y + 17 = 0$